6) **Reši enačbo!**

$$\arctg(x^2-3x-3)=\frac{\pi}{4}$$

$$\tg\frac{\pi}{4}=x^2-3x-3$$

$$1=x^2-3x-3$$

$$x^2-3x-4=0$$

$$(x-4)(x+1)=0$$

$$x_1=4,\ x_2=-1$$

$$\arctg(x^2-3x-3)=\frac{\pi}{4}\quad /\cdot\tg$$

$$\tg(\arctg(x^2-3x-3))=\tg\frac{\pi}{4}$$

$$x^2-3x-3=1$$

Trigonometrične enačbe

13.1.$\overline{03}$

- To so enačbe v katerih nastopajo kotne funkcije, neznanka pa je kot (argument) kotne funkcije.

Primeri:

① $2\cdot\cos x=1$

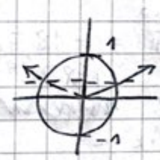
② $\tg(x+\frac{\pi}{3})=\sqrt{3}$

③ $\sin^2 x+5\cos x+2=0$

④ $2\tg x-3\ctg x=0$ ...

iščemo vse rešitve

① $\sin x=a$; $-1\leq a\leq 1$



$\sin x=3$, ni rešitve

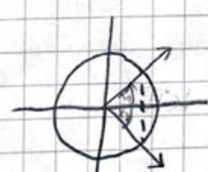
perioda

$$\boxed{\begin{aligned}x_1&=\arcsin a+2k\pi\\ x_2&=(\pi-x_1)+2k\pi\end{aligned}}$$

suplementarna

$k\in\mathbb{Z}$

② $\cos x=a$; $-1\leq a\leq 1$



$$\boxed{\begin{aligned}x_1&=\arccos a+2k\pi\\ x_2&=-x_1+2k\pi\end{aligned}}$$

$k\in\mathbb{Z}$

③ $\tg x=a$; - enačba je vedno rešljiva, 1 reš.

$$\boxed{x=\arctg a+k\pi}$$

$k\in\mathbb{Z}$

(4) ctg x = a

-kotangens spremenim v tg in nato rešim enačbo!

$\mathrm{ctg}\, x = \mathrm{tg}(90° - x)$

$\mathrm{ctg}\, x = \frac{1}{\mathrm{tg}\, x}$

I. 1.) $\cos x = -\frac{1}{2}$

$x_1 = \arccos\left(-\frac{1}{2}\right) + 2k\pi$

$x_1 = \frac{2\pi}{3} + 2k\pi \quad ; k \in \mathbb{Z}$

$\frac{2\pi}{3} = 120°$

$x_2 = -\frac{2\pi}{3} + 2k\pi \quad ; k \in \mathbb{Z}$

$-\frac{2\pi}{3} = -120°$

2.) $\sin x = 0{,}7869$

$x_1 = \arcsin(0{,}7869) + 2k\pi = 0{,}906 + 2k\pi$ ali $51{,}89° + k \cdot 360°$

$x_2 = \underbrace{(\pi - 0{,}906)}_{180° - x_1} + 2k\pi = 2{,}236 + 2k\pi$ ali $128{,}10° + k \cdot 360°$

$k \in \mathbb{Z}$

3.) $\sin x = -0{,}321$

$x_1 = \arcsin(-0{,}321) + \overset{k \cdot 360°}{2k\pi} = -18{,}72° + k \cdot 360°$

$x_2 = (180° - x_1) + 2k\pi = 198{,}72° + k \cdot 360°$

$(180° - (-18{,}72°)) + 360° \cdot k =$

$\Big\}\ k \in \mathbb{Z}$

4.) $\mathrm{tg}\left(\underbrace{x + \frac{\pi}{3}}_{= z}\right) = \sqrt{3}$

za kot uvedemo novo neznanko $z = x + \frac{\pi}{3}$

$\mathrm{tg}\, z = \sqrt{3}$

$z = \mathrm{arctg}\sqrt{3} + k\pi$

$z = \frac{\pi}{3} + k\pi$

$z = x + \frac{\pi}{3}$

$x = z - \frac{\pi}{3} = \cancel{\frac{\pi}{3}} + k\pi - \cancel{\frac{\pi}{3}} = k\pi \qquad k \in \mathbb{Z}$

Učb. str. 57/1.n)

4 rešitve!

$\boxed{k \in \mathbb{Z}}$ ↓ NAPIŠI VSAKIČ PRI NALOGAH

$z = 3x - \pi$

$\cos^2(3x-\pi) = \frac{1}{2}$

$\cos^2(3x-\pi) - \frac{1}{2} = 0$

$\left(\cos(3x-\pi) - \frac{1}{\sqrt{2}}\right)\left(\cos(3x-\pi) + \frac{\sqrt{2}}{2}\right) = 0$

$\cos z = \frac{\sqrt{2}}{2}$

$z_1 = \arccos\frac{\sqrt{2}}{2} + 2k\pi = \frac{\pi}{4} + 2k\pi$ ✓

$z_1 = 3x_1 - \pi$

$3x_1 = z_1 + \pi$

$x_1 = \frac{z_1+\pi}{3} = \frac{\frac{\pi}{4} + 2k\pi + \pi}{3} = \frac{\frac{\pi}{4} + \frac{4\pi}{4} + \frac{8k\pi}{4}}{3} = \frac{\frac{5\pi + 8k\pi}{4}}{\frac{3}{1}}$

$x_1 = \frac{5\pi + 8k\pi}{12} = \frac{5\pi}{12} + \frac{2}{3}k\pi$

$z_2 = -z_1 + 2k\pi = -\frac{\pi}{4} + 2k\pi$ ✓

$x_2 = \frac{z_2 + \pi}{3} = \frac{-\frac{\pi}{4} + 2k\pi + \pi}{3}$

$x_2 = \frac{-\frac{\pi}{4} + \frac{4\pi}{4} + \frac{8k\pi}{4}}{3} = \frac{\frac{3\pi + 8k\pi}{4}}{\frac{3}{1}} = \frac{3\pi + 8k\pi}{12}$

$x_2 = \frac{\pi}{4} + \frac{2}{3}k\pi$

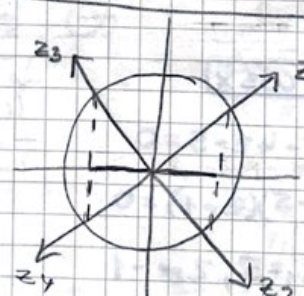
$\cos z = -\frac{\sqrt{2}}{2}$

$z_3 = \arccos\left(-\frac{\sqrt{2}}{2}\right) + 2k\pi$

$z_3 = \frac{3\pi}{4} + 2k\pi$ ✓

$z_4 = -z_3 + 2k\pi$

$z_4 = -\frac{3\pi}{4} + 2k\pi$ ✓



$z = (3x - \pi)$

$z = \frac{\pi}{4} + k \cdot \frac{\pi}{2}$ → združim rešitve v eno

$\frac{\pi}{4} + k \cdot \frac{\pi}{2} = 3x - \pi \quad / \cdot 4$

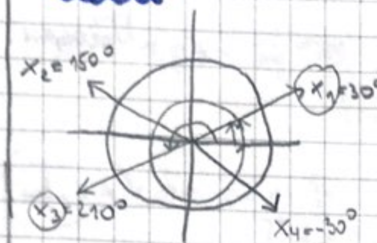
$\pi + 2k\pi = 12x - 4\pi$

$12x = 5\pi + 2k\pi \quad / : 12$

$x = \frac{5\pi}{12} + \frac{2k\pi}{12} = \frac{\pi}{12}(5 + 2k)$

Rešitve pri trigonometričnih enačbah se lahko tudi združijo:

Zgled:



$$\left.\begin{array}{l} x_1 = 30° + k\cdot 360° \\ x_2 = 150° + k\cdot 360° \\ x_3 = 210° + k\cdot 360° \\ x_4 = -30° + k\cdot 360° \end{array}\right) \quad k \in \mathbb{Z}$$

$\ldots\ 30°,\ 390°,\ 750°\ \ldots$

$\ldots\ -210°,\ 150°,\ 510°\ \ldots$

$\ldots\ -150°,\ 210°,\ 570°\ \ldots$

$\ldots\ -30°,\ 330°,\ 690°\ \ldots$

$x_1 = 30° + k\cdot 180°$ $(x_1 \,\&\, x_3)$

$x_2 = -30° + k\cdot 180°$ $(x_2 \,\&\, x_4)$

II.

2. č)

$$\sin^2 x + 4\cdot\cos x + 4 = 0$$

→ Kadar v trigonometrični enačbi nastopajo različne kotne funkcije, jih pretvorimo (s pomočjo zvez) v eno samo kotno funkcijo.

$1 - \cos^2 x + 4\cdot\cos x + 4 = 0 \quad /\cdot(-1)$ $\quad \sin^2 x + \cos^2 x = 1$

$\cos^2 x - 4\cdot\cos x - 5 = 0$

kvadr. enačba

rešimo lahko tudi z razstavljanjem

$(\cos x - 5)(\cos x + 1) = 0$

$\boxed{z = \cos x}$

$z^2 - 4z - 5 = 0$

$(z-5)(z+1) = 0$

$z_1 = 5,\ z_2 = -1$

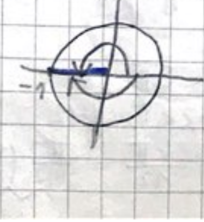
① $\cos x = 5$ ② $\cos x = -1$

– ni rešitve (ni takšnega kota, katerega cos je 5)

$\boxed{x_1 = \pi + 2k\pi}$ $\quad k \in \mathbb{Z}$

$\cancel{x_2 = -\pi + 2k\pi}$

rešitvi sta enaki



2i

$\sin^3 x - \sin^2 x = 9(\sin x - 1)$

$\sin^3 x - \sin^2 x - 9(\sin x - 1) = 0$

$\sin^3 x - \sin^2 x - 9\sin x + 9 = 0$

$\sin^2 x(\sin x - 1) - 9(\sin x - 1) = 0$

$(\sin^2 x - 9)(\sin x - 1)^2 = 0$

$(\sin x - 3)(\sin x + 3)(\sin x - 1)^2 = 0$

1.) $\sin x - 3 = 0$
$\sin x = +3$ ni rešitve

2.) $\sin x + 3 = 0$
$\sin x = -3$ ni rešitve

3.) $\sin x - 1 = 0$
$\sin x = 1$

$k \in \mathbb{Z}$

$x_1 = \arcsin 1 + 2k\pi = \frac{\pi}{2} + 2k\pi$

$x_2 = \pi - \frac{\pi}{2} + 2k\pi = \frac{\pi}{2} + 2k\pi = x_1$

$x = \frac{\pi}{2} + 2k\pi = \frac{\pi + 4k\pi}{2} = \underline{\underline{\frac{\pi}{2}(1+4k)}}$ 2 $x = (2k+1)\frac{\pi}{2}$ ✓

Učb. str. 58/2e

$\sin x + \cos 2x = 1$

$\sin x + \cos^2 x - \sin^2 x - 1 = 0 \quad / \cdot (-1)$

$1 - \cos^2 x + \sin^2 x - \sin x = 0$

$\sin^2 x + \sin^2 x - \sin x = 0$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

REŠITVE ✓

$$\boxed{\begin{aligned} x_1 &= k\pi \\ x_2 &= \frac{\pi}{6}(12k+1) \\ x_3 &= \frac{\pi}{6}(12k+5) \end{aligned}}$$

1.) $\sin x = 0$

$x_1 = \arcsin 0 + 2k\pi =$
$= 0 + 2k\pi = \underline{2k\pi}$

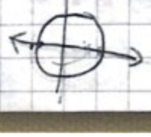
$x_2 = \pi - \arcsin 0 + 2k\pi =$
$= \pi - 0 + 2k\pi =$
$= \underline{\pi + 2k\pi}$

2.) $2\sin x - 1 = 0$

$2\sin x = 1 \quad / : 2$
$\sin x = \frac{1}{2}$

$x_3 = \arcsin\frac{1}{2} + 2k\pi = \underline{\frac{\pi}{6} + 2k\pi}$

$x_4 = \frac{6\pi}{6} - \frac{\pi}{6} + 2k\pi = \underline{\frac{5\pi}{6} + 2k\pi}$



$x = \underline{k\pi}$ ✓

$\boxed{k \in \mathbb{Z}}$

f) $\mathrm{tg}\,x - 6\,\mathrm{ctg}\,x = 1$

$\mathrm{tg}\,x - 6\frac{1}{\mathrm{tg}\,x} - 1 = 0 \;/\cdot \mathrm{tg}\,x$

$\mathrm{tg}^2 x - 6 - \mathrm{tg}\,x = 0$

$\mathrm{tg}^2 x - \mathrm{tg}\,x - 6 = 0$

$(\mathrm{tg}\,x - 3)(\mathrm{tg}\,x + 2) = 0$

1) $\mathrm{tg}\,x - 3 = 0$

$\mathrm{tg}\,x = 3$

$x_1 = \mathrm{arctg}\,3 + k\pi =$

$= 1{,}249 + k\pi$

$= 71°34' + 180°k$

2.) $\mathrm{tg}\,x + 2 = 0$

$\mathrm{tg}\,x = -2$

$x_2 = \mathrm{arctg}(-2) + k\pi =$

$= -1{,}107 + k\pi$

$= -63°26' + k \cdot 180°$

III. homogene trigonometrične enačbe

→ so enačbe, kjer so vsi členi enake stopnje.

1. $3\sin x - 5\cos x = 0$
2. $3\sin^2 x - 5\sin x\cos x + 2\cos^2 x = 0$
3. $3\sin^2 x + \cos 2x = 0$ (š ni homogena)

↓ Enačba se da pretvoriti v homogeno

Vsako homog. trig. enačbo spremenimo v tg; enačbo delimo z $\cos x$ ali $\cos^2 x$.

→ zato to ni potrebno pisat

$3\sin x - 5\cos x = 0 \;/ : \cos x \,(\neq 0)$ – sin in cos ne moreta biti hkrati enaka nič!

$3\,\mathrm{tg}\,x - 5 = 0$

$3\,\mathrm{tg}\,x = 5 \;/ : 3$

$\mathrm{tg}\,x = \frac{5}{3}$

$x = \mathrm{arctg}\frac{5}{3} + k\pi$

$x = 1{,}030 + k\pi$

učb. str. 58 / 3.g)

$2\cos^2 x + \sin 2x = 2$

$2\cos^2 x + 2\cdot\sin x\cdot\cos x - 2 = 0 \quad /:2$

$\cos^2 x + \sin x\cdot\cos x - 1 = 0$

$\cos^2 x + \sin x\cdot\cos x - (\sin^2 x + \cos^2 x) = 0 \quad /:\cos^2 x$ enačba je homogena

$\cancel{1} + \operatorname{tg} x - \operatorname{tg}^2 x - \cancel{1} = 0 \quad /\cdot(-1)$

$\operatorname{tg}^2 x - \operatorname{tg} x = 0$

$\operatorname{tg} x(\operatorname{tg} x - 1) = 0$

1.) $\operatorname{tg} x = 0$

$x_1 = \operatorname{arctg} 0 + k\pi$

$= \underline{k\pi}$ ✓

2.) $\operatorname{tg} x - 1 = 0$

$\operatorname{tg} x = 1$

$x_2 = \operatorname{arctg} 1 + k\pi =$

$= \underline{\frac{\pi}{4} + k\pi} = \frac{\pi}{4} + \frac{4k\pi}{4} = \underline{\frac{\pi}{4}(1+4k)}$ ✓

3.e) $\sin^2 x + \sqrt{3}\cdot\sin 2x + 3\cdot\cos^2 x = 0$

$\sin^2 x + \sqrt{3}\cdot 2\sin x\cdot\cos x + 3\cdot\cos^2 x = 0 \quad /:\cos^2 x$

$\operatorname{tg}^2 x + 2\sqrt{3}\cdot\operatorname{tg} x + 3 = 0$

$(\operatorname{tg} x + \sqrt{3})(\operatorname{tg} x + \sqrt{3}) = 0$

$\operatorname{tg} x + \sqrt{3} = 0 \qquad \underline{\operatorname{tg} x = -\sqrt{3}}$

$x = \operatorname{arctg}(-\sqrt{3}) + k\pi = \underline{-\frac{\pi}{3} + k\pi} = \underline{\frac{\pi}{3}(3k-1)}$ ✓

IV. reševanje s polovičnimi koti

učb. str. 57 / z 9

$2\cdot\cos x + \sin x = 1$

$2\cdot\cos 2\cdot\frac{x}{2} + \sin 2\cdot\frac{x}{2} = 1$

$2\cdot(\cos^2(\frac{x}{2}) - \sin^2(\frac{x}{2})) + 2\cdot\sin(\frac{x}{2})\cdot\cos(\frac{x}{2}) = 1$

$2\cos^2\frac{x}{2} - 2\cdot\sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cdot\cos\frac{x}{2} = \sin^2\frac{x}{2} + \cos^2\frac{x}{2}$

→ enačba je homogena

$\cos^2\frac{x}{2} - 3\cdot\sin^2\frac{x}{2} + 2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2} = 0 \quad /:\cos^2\frac{x}{2} \neq 0$

$1 - 3\operatorname{tg}^2(\frac{x}{2}) + 2\cdot\operatorname{tg}\frac{x}{2} = 0 \quad /\cdot(-1)$

$3\operatorname{tg}^2(\frac{x}{2}) - 2\cdot\operatorname{tg}(\frac{x}{2}) - 1 = 0 \qquad \boxed{t = \operatorname{tg}\frac{x}{2}}$

$3t^2-2t-1=0$

$(3t+1)(t-1)=0$

$3t+1=0 \qquad t-1=0$

$t=-\frac{1}{3} \qquad t=1$

① $tg\frac{x}{2}=-\frac{1}{3}$ ali ② $tg\frac{x}{2}=1$

$\frac{x_1}{2}=arctg(-\frac{1}{3})+k\pi \;/\cdot 2 \qquad \frac{x_2}{2}=arc\,tg\,1+k\pi \;/\cdot 2$

$x_1=-36,87°+2k\pi \qquad x_2=90°+2k\pi$