6. Pokaži, da funkcija $y=A\cdot e^{x}+B\cdot e^{-x}$ ustreza enačbi $y''=y$.

$y'=A\cdot e^{x}+B(-1)e^{-x}=Ae^{x}-Be^{-x}$

$y''=Ae^{x}-B(-1)e^{-x}=Ae^{x}+Be^{-x}=y$ ✓

7. Pokaži, da funkcija $y=A\cdot\sin x+B\cdot\cos x$ ustreza enačbi $y''+y=0$.

$y'=A\cdot\cos x-B\cdot\sin x$

$y''=-A\cdot\sin x-B\cdot\cos x$

$-A\cdot\sin x-B\cdot\cos x+A\cdot\sin x+B\cdot\cos x\stackrel{?}{=}0$

$0=0$ ✓

→ $y''=\dfrac{3x\cdot\left(-\frac{(2x+y)}{x+2y}\right)-3y}{(x+2y)^2}=\dfrac{\frac{-6x^2-3xy-3xy-6y^2}{(x+2y)}}{(x+2y)^2}$

$y''=\dfrac{-6x^2-6xy-6y^2}{(x+2y)^3}=\dfrac{-6(x^2+xy+y^2)}{(x+2y)^3}=\dfrac{-6a^2}{(x+2y)^3}$

Učb. str. 114/4. Izračunaj odvode funkcij v dani točki T:

a) $y=x^2\cdot\ln x$, $T(3,y)$

$y'=2x\cdot\ln x+x^2\cdot\frac{1}{x}$

$y'=2x\ln x+x$

$y'(3)=2\cdot3\ln3+3$

$y'(3)=6\ln3+3$

$y'(3)=3(2\ln3+1)$

$y'(3)=3(\ln9+1)$ $\stackrel{?}{=}$ $3\cdot\ln(9e)$

b) $y=e^{x^2}$, $T(1,y)$

$y'=2x\cdot e^{x^2}$

$y'(1)=2e$

c) $y=x\cdot\sin x$, $T(\pi/4,y)$

$y'=\sin x+x\cdot\cos x$

$y'\left(\frac{\pi}{4}\right)=\sin\frac{\pi}{4}+\frac{\pi}{4}\cdot\cos\frac{\pi}{4}$

$y'\left(\frac{\pi}{4}\right)=\frac{\sqrt2}{2}+\frac{\pi}{4}\cdot\frac{\sqrt2}{2}=\frac{\sqrt2}{8}(4+\pi)$

č) $y = \ln \operatorname{ctg} x$, $T(\pi/4, y)$

$z = \operatorname{ctg} x$, $y = \ln z$

$y' = z' \cdot \frac{1}{z}$, $z' = -\frac{1}{\sin^2 x}$

$y' = -\frac{1}{\sin^2 x} \cdot \frac{1}{\operatorname{ctg} x} = -\frac{1}{\sin^2 x} \cdot \frac{\sin x}{\cos x} = -\frac{1}{\sin x \cdot \cos x}$

$y'(\pi/4) = -\frac{1}{\sin\frac{\pi}{4} \cdot \cos\frac{\pi}{4}} = -\frac{1}{\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2}} = -\frac{4}{2} = -2$

DOMAČA NALOGA

16.2.04

① Izračunaj kot (na stotinko stopinje) pod katerim se sekata krivulji $2x^2 - y^2 = 5$ in $x^2 + y^2 = 4$ v drugem kvadrantu. Graf!

HIPERBOLA: $2x^2 - y^2 = 5 \; /:5$

$\frac{x^2}{\frac{5}{2}} - \frac{y^2}{5} = 1$ $S(0,0)$, $a = \sqrt{\frac{5}{2}}$, $b = \sqrt{5}$

KROŽNICA: $x^2 + y^2 = 4$ $S(0,0)$, $r = 2$

Presečišča:

$2x^2 - y^2 = 5$
$x^2 + y^2 = 4$) +

$3x^2 = 9 \; /:3$
$x^2 = 3$
$x_1 = \sqrt{3}$, $x_2 = -\sqrt{3}$

$y^2 = 4 - x^2$

1.) $x = \sqrt{3}$
$y^2 = 4 - 3$
$y^2 = 1$
$y_1 = 1$, $y_2 = -1$

2.) $x = -\sqrt{3}$
$y^2 = 4 - 3$
$y^2 = 1$
$y_1 = 1$, $y_2 = -1$

$P_1(\sqrt{3}, 1)$, $P_2(\sqrt{3}, -1)$, $P_3(-\sqrt{3}, 1)$, $P_4(-\sqrt{3}, -1)$

Kot: ~ $P_3(-\sqrt{3}, 1)$

$\operatorname{tg} \varphi = \left| \frac{k_2 - k_1}{1 + k_1 k_2} \right|$

$2x^2 - y^2 = 5$ $kt_1 = y'(P_3)$

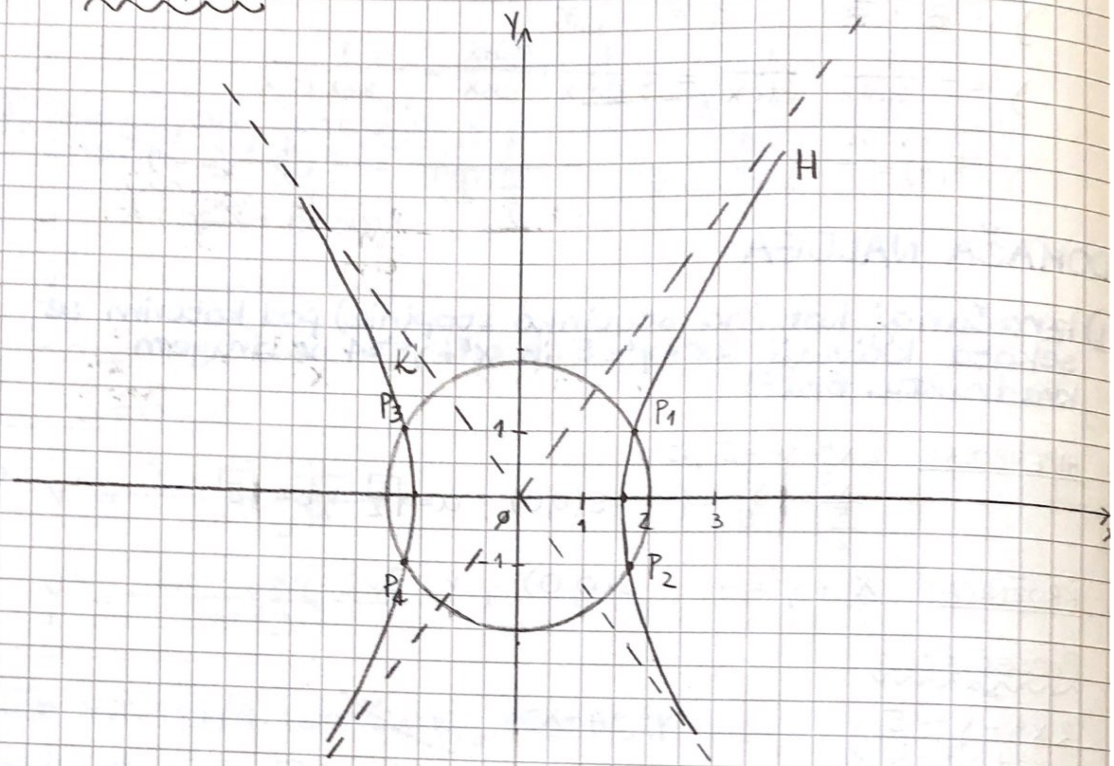
odvod: $4x - 2y \cdot y' = 0 \; /:2$
$y \cdot y' = 2x$
$y' = \frac{2x}{y}$, $kt_1 = \frac{2x_1}{y_1} = \frac{-2(\sqrt{3})}{1} = -2\sqrt{3}$

$x^2 + y^2 = 4$, $kt_2 = y'(P_3)$

odvod: $2x + 2y \cdot y' = 0 \; /:2$
$y \cdot y' = -x$
$y' = -\frac{x}{y}$, $kt_2 = \frac{-x_1}{y_1} = \frac{-(-\sqrt{3})}{1} = \sqrt{3}$

$$\operatorname{tg}\varphi = \left|\frac{\sqrt{3}-(-2\sqrt{3})}{1-2\sqrt{3}\cdot\sqrt{3}}\right| = \left|\frac{3\sqrt{3}}{1-6}\right| = \left|\frac{3\sqrt{3}}{-5}\right| = \frac{3\sqrt{3}}{5}$$

$\varphi = 46{,}10°$



② Izračunaj enačbo tangente in normale na krivuljo $y = x^3 + 3\sqrt[3]{x}$ v točki $T_1(-1, y)$.

tangenta: $y - y_1 = k_t(x - x_1)$

$T_1(-1, y)$, $y_1 = (-1)^3 + 3\sqrt[3]{-1} = -1 + 3(-1) = -4 \rightarrow T_1(-1, -4)$

$k_t = y'(x_1)$

$y' = 3x^2 + 3\cdot\frac{1}{3}x^{-\frac{2}{3}} = 3x^2 + \frac{1}{\sqrt[3]{x^2}}$

$k_t = 3(-1)^2 + \frac{1}{\sqrt[3]{(-1)^2}} = 3 + 1 = 4$

t: $y + 4 = 4(x + 1)$

$y = 4x + 4 - 4$

$y = 4x$

normala: $y - y_1 = k_n(x - x_1)$

$k_n = -\frac{1}{y'(x_1)}$ $\qquad y + 4 = -\frac{1}{4}(x + 1)$

$k_n = -\frac{1}{4}$ $\qquad y = -\frac{1}{4}x - \frac{1}{4} - 4$

$y = -\frac{1}{4}x - \frac{17}{4}$

③ Izračunaj odvod:

a) $y = x + x^{-1}$ po pravilu in po definiciji

po definiciji:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$y' = \lim_{h \to 0} \frac{x + h + (x+h)^{-1} - x - x^{-1}}{h} = \lim_{h \to 0} \frac{h + \frac{1}{x+h} - \frac{1}{x}}{h} =$$

$$= \lim_{h \to 0} \left(\frac{\frac{hx^2 + h^2x + x - x - h}{x(x+h)}}{\frac{h}{1}} \right) = \lim_{h \to 0} \frac{h(x^2 + hx - 1)}{x(x+h) \cdot h} = \frac{x^2 - 1}{x^2} = 1 - \frac{1}{x^2}$$

po pravilu:

$$y' = 1 - 1x^{-2} = 1 - \frac{1}{x^2}$$

b) $y = e^{2x} \cdot \cos(3x)$

$$y' = 2 \cdot e^{2x} \cdot \cos 3x + e^{2x} \cdot (-\sin(3x)) \cdot 3$$

$$y' = e^{2x}(2 \cdot \cos 3x - 3 \sin 3x)$$

c) $y = \sqrt{\ln(x^2 - 1)}$

$y = z^{\frac{1}{2}}$ $\qquad z = \ln(x^2 - 1)$

$\qquad z' = 2x \cdot \frac{1}{x^2 - 1} = \frac{2x}{x^2 - 1}$

$$y' = \frac{1}{2} z^{-\frac{1}{2}} \cdot z'$$

$$y' = \frac{1}{2\sqrt{\ln(x^2-1)}} \cdot \frac{2x}{x^2 - 1} = \frac{x}{(x^2 - 1)\sqrt{\ln(x^2 - 1)}}$$

4. V katerih točkah ima funkcija $y = x + \sin x$ tangento z naklonskim kotom 45°?

$\operatorname{tg} \varphi = k_t \qquad \operatorname{tg} 45 = k_t = 1$

$k_t = y'(x_1)$

$y' = 1 + \cos x$

$1 + \cos x = 1$

$\cos x = 0$

$x_1 = \frac{\pi}{2} + k\pi, \; k \in \mathbb{Z}$

$y_1 = \frac{\pi}{2} + k\pi \pm 1, \; k \in \mathbb{Z}$

→ v točkah:

$T\left(\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi \pm 1\right)$

$k \in \mathbb{Z}$

(+1 za sode k
−1 za lihe k)

1. Načrtaj graf polinoma $p(x) = -x^3 + 3x + 2$ ter poišči točko, kjer je tangenta na graf polinoma vzporedna s premico $y = -3x + 5$.

$p(x) = -x^3 + 3x + 2$ (graf)

- ničle:

	-1	0	3	2
		1	-1	-2
-1	-1	1	2	0

$x_1 = -1$

$-x^2 + x + 2 = 0 \quad / \cdot (-1)$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x_2 = 2, \; x_3 = -1$

NIČLE: $x_1 = -1$ (2. st.)

$x_2 = 2$ (1. st.)

- zač. vr.:

$x = 0, \; p(x) = 2 \quad A(0, 2)$

- potek funkcije pri velikih x-ih:

$x \mapsto +\infty, \; y \to -\infty$

$x \mapsto -\infty, \; y \to +\infty$

- ekstremi:

$y' = 0, \; p'(x) = -3x^2 + 3, \; -3x^2 + 3 = 0 \quad / : (-3)$

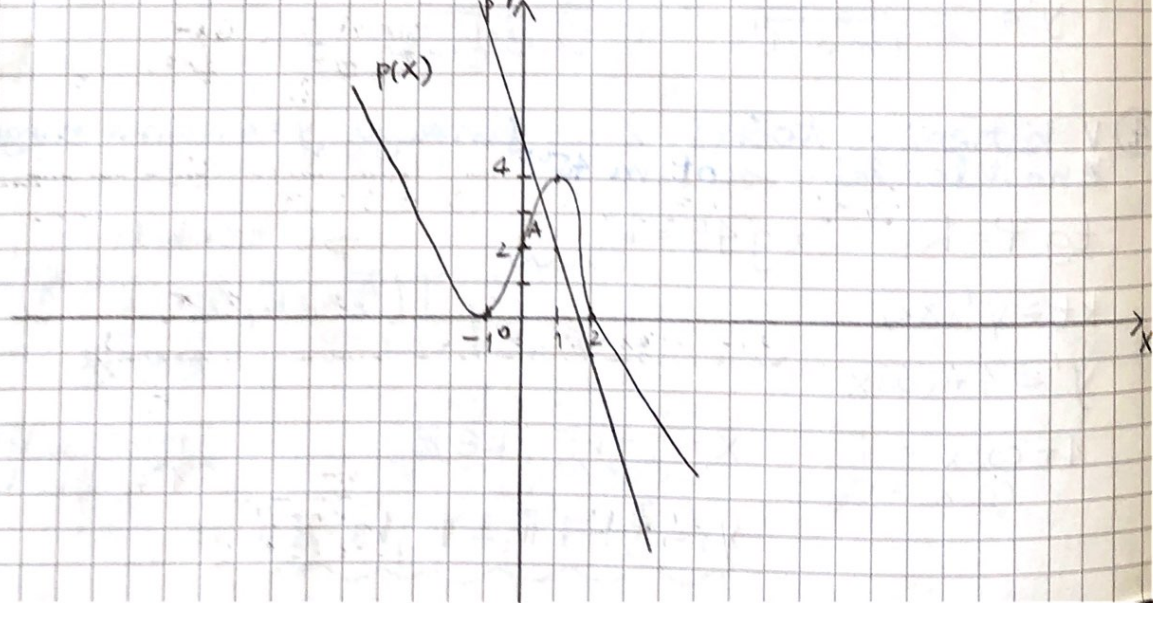
$x^2 - 1 = 0$

$(x-1)(x+1) = 0$

$x_1 = 1, \; x_2 = -1$

$y_1 = -1 + 3 + 2 = 4 \quad T_1(1, 4)$

$y_2 = 1 - 3 + 2 = 0 \quad T_2(-1, 0)$



$t \parallel p \Leftrightarrow k_t = k_p$

$p: y = -3x + 5 \quad , k_p = -3 \Rightarrow k_t = -3$

$k_t = y'(x_1)$

$-3 = -3x^2 + 3$

$3x^2 = 6 \quad /:3$

$x^2 = 2$

$y_1 = -(\sqrt{2})^3 + 3\sqrt{2} + 2 = -\sqrt{8} + 3\sqrt{2} + 2$

$y_2 = -(-\sqrt{2})^3 + 3(-\sqrt{2}) + 2 = \sqrt{8} - 3\sqrt{2} + 2$

$x_1 = \sqrt{2}, \; x_2 = -\sqrt{2}$

$T_1(\sqrt{2}, -\sqrt{8} + 3\sqrt{2} + 2)$

$T_2(-\sqrt{2}, \sqrt{8} - 3\sqrt{2} + 2)$

(2) Dana je funkcija $f(x) = \frac{2x-1}{x+a}$, katere graf gre skozi točko $A(-2,3)$.

Določi $f(x)$, izračunaj odvod funkcije po definiciji in pravilu ter ugotovi sodost oz. lihost te funkcije!

A: $3 = \frac{2(-2)-1}{-2+a} \quad / \cdot (a-2)$

$3(a-2) = -4-1$

$3a - 6 = -5$

$3a = 1$

$a = 1/3$

$f(x) = \frac{2x-1}{x+\frac{1}{3}}$

$f(x) = \frac{\frac{2x-1}{1}}{\frac{3x+1}{3}} = \frac{3(2x-1)}{3x+1}$

$f(x) = \frac{6x-3}{3x+1}$

odvod po pravilu:

$$f'(x) = \frac{(6x-3)'(3x+1) - (6x-3)(3x+1)'}{(3x+1)^2} = \frac{6(3x+1) - (6x-3) \cdot 3}{(3x+1)^2}$$

$$f'(x) = \frac{18x + 6 - 18x + 9}{(3x+1)^2} = \frac{15}{(3x+1)^2}$$

odvod po definiciji:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{6(x+h)-3}{3(x+h)+1} - \frac{6x-3}{3x+1}}{h} =$$

$$= \lim_{h \to 0} \frac{\frac{6x+6h-3}{3x+3h+1} - \frac{6x-3}{3x+1}}{h} = \lim_{h \to 0} \frac{\frac{18x^2 + 18hx - 9x + 6x + 6h - 3 - (6x-3)(3x+3h+1)}{(3x+1)(3x+3h+1)}}{\frac{h}{1}} =$$

$$= \lim_{h \to 0} \frac{\frac{18x^2 + 18hx - 3x + 6h - 3 - 18x^2 + 9x - 18hx + 9h - 6x + 3}{(3x+1)(3x+3h+1)}}{\frac{h}{1}} =$$

$$= \lim_{h \to 0} \frac{15h}{h(3x+1)(3x+3h+1)} = \frac{15}{(3x+1)^2}$$

$f(x) = \frac{6x-3}{3x+1}$

sodost:

$f(-x) = f(x)$

lihost:

$f(-x) = -f(x)$

$f(-x) = \frac{-6x-3}{-3x+1} = \frac{-(6x+3)}{-(3x-1)} = \frac{6x+3}{3x-1}$

$-f(x) = -\frac{6x-3}{3x+1} = \frac{-6x+3}{3x+1}$

$f(-x) \neq f(x)$, $f(-x) \neq -f(x)$ } funkcija ni ne soda, ne liha

③ Izračunaj odvode:

a) $y = \ln\cos x - \frac{1}{2}\cos^2 x$

$z = \cos x \qquad z' = -\sin x$

$y = \ln z - \frac{1}{2} z^2$

$y' = \frac{1}{z} \cdot z' - z \cdot z'$

$y' = \frac{1}{\cos x} \cdot (-\sin x) - \cos x \, (-\sin x)$

$y' = -\frac{\sin x}{\cos x} + \cos x \cdot \sin x \qquad -\frac{\sin^2 x}{\cos x}$

$y' = -\operatorname{tg} x + \cos x \cdot \sin x$

b) $y^2 + 2y + 2x - 4 = 0$

odvod: $2y \cdot y' + 2y' + 2 = 0 \quad /:2$

$y \cdot y' + y' = -1$

$y'(y+1) = -1$

$y' = -\frac{1}{y+1}$

④ Zapiši enačbo tangente na krivuljo $f(x) = 2x \cdot \cos x$ v točki z absciso 2π.

tangenta: $y - y_1 = k_t (x - x_1) \qquad T(x_1, y_1) \; ; \; x_1 = 2\pi$

$y_1 = 2 \cdot 2\pi \cdot \cos 2\pi = 4\pi \cdot 1 = 4\pi \rightarrow T(2\pi, 4\pi)$

$k_t = y'(x_1)$

$y' = (2x)' \cdot \cos x + 2x (\cos x)' = 2\cos x - 2x \cdot \sin x$

$k_t = y'(2\pi) = 2 \cdot \cos 2\pi - 2x \cdot \sin 2\pi = 2$

(t:) $y - 4\pi = 2(x - 2\pi)$

$y = 2x - 4\pi + 4\pi \qquad y = 2x$

1. Izračunaj odvod!

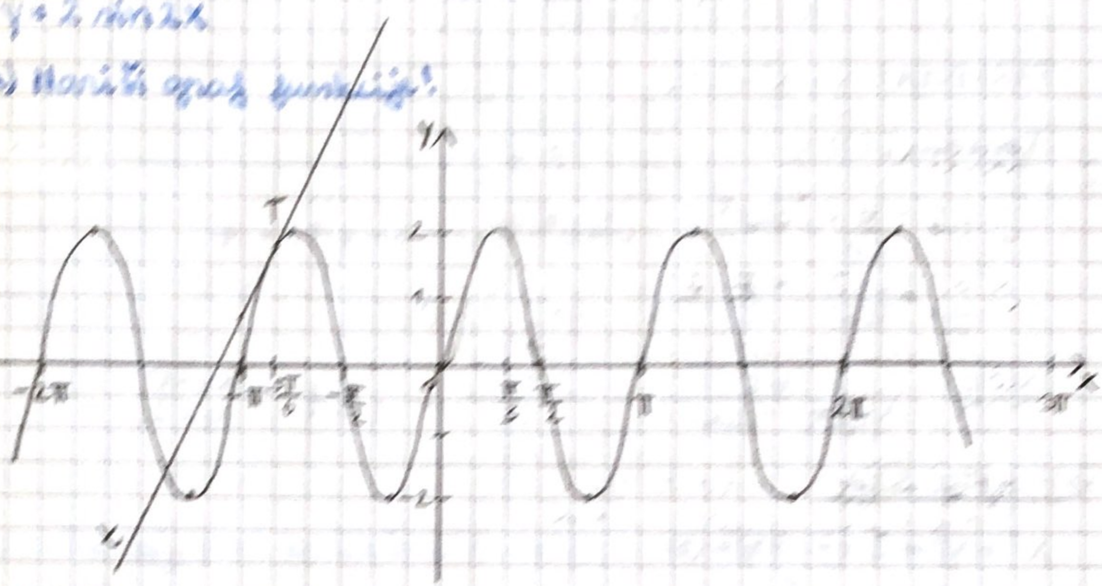
$y = \sqrt[2]{\frac{x^3 \sqrt[4]{x}}{x \sqrt[3]{x}}} = \frac{x^{\frac{3}{2}} \cdot x^{\frac{1}{8}}}{x^{\frac{1}{2}} \cdot x^{\frac{1}{6}}} = x^{\frac{3}{2}+\frac{1}{8}-\frac{1}{2}-\frac{1}{6}} = x^{1+\frac{1}{8}-\frac{1}{6}}$

$y = x^{\frac{12+1-2}{12}} = x^{\frac{13}{12}}$

$y' = \frac{13}{12} x^{-\frac{1}{12}} = \frac{13}{12\sqrt[12]{x}}$

2. $y = 2 \sin 2x$

a) Nariši graf funkcije!



b) Določi periodo!

$\frac{2\pi}{\omega}$ = osnovna perioda $\qquad y = 2 \sin \underset{\omega}{2} x$

$\frac{2\pi}{2} = \pi$ $\qquad$ osnovna perioda: π

c) Zapiši enačbo tangente na krivuljo v točki $T_1\left(-\frac{5\pi}{6}, y_1\right)$! Tangento nariši!

<u>tangenta:</u> $y - y_1 = k_t(x - x_1)$

$y_1 = 2 \sin 2\left(-\frac{5\pi}{6}\right) = 2 \sin\left(-\frac{5\pi}{3}\right) = 2 \sin(-300) = 1{,}73$ $\quad (\sqrt{3})$

$k_t = y'(x_1)$

$y' = 2 \cdot \cos 2x \cdot 2 = 4 \cdot \cos 2x$

$k_t = 4 \cdot \cos 2\left(-\frac{5\pi}{6}\right) = 4 \cos\left(\frac{5\pi}{3}\right) = 2$

t) $y - 2 \cdot \sin\left(-\frac{5\pi}{3}\right) = 2\left(x + \frac{5\pi}{6}\right)$

$y = 2x + \frac{5\pi}{3} + \sqrt{3} = 2x + \frac{5\pi + 3\sqrt{3}}{3}$

$$y = 2x + \frac{5\pi + 3\sqrt{3}}{3}$$

④ Dani sta enačbi $x^2 + y^2 - 2x + 4y = 0$ in $y = x^2 - 2x$.

a) Nariši krivulji in izračunaj presečišča.

KROŽNICA: $x^2 - 2x + y^2 + 4y = 0$

$(x-1)^2 - 1 + (y+2)^2 - 4 = 0$

$(x-1)^2 + (y+2)^2 = 5$

$S(1, -2)$, $r = \sqrt{5}$

PARABOLA: $y = x^2 - 2x$

Teme: $T(p, q)$

$p = \frac{-b}{2a} = \frac{2}{2} = 1$, $D = b^2 - 4ac = 4 - 4 \cdot 1 \cdot 0 = 4$

$q = \frac{-D}{4a} = \frac{-4}{4} = -1$ $\rightarrow T(1, -1)$

Ničle: $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{+2 \pm 2}{2} = 1 \pm 1$ $\quad x_1 = 0$, $x_2 = 2$

→ PRESEČIŠČA

$x^2 + y^2 - 2x + 4y = 0$

$y = x^2 - 2x$

$x^2 + (x^2 - 2x)^2 - 2x + 4(x^2 - 2x) = 0$

$x^2 + x^4 - 4x^3 + 4x^2 - 2x + 4x^2 - 8x = 0$

$x^4 - 4x^3 + 9x^2 - 10x = 0$

$x(x^3 - 4x^2 + 9x - 10) = 0$

$x_1 = 0$

	1	−4	9	−10
		2	−4	10
2	1	−2	5	0

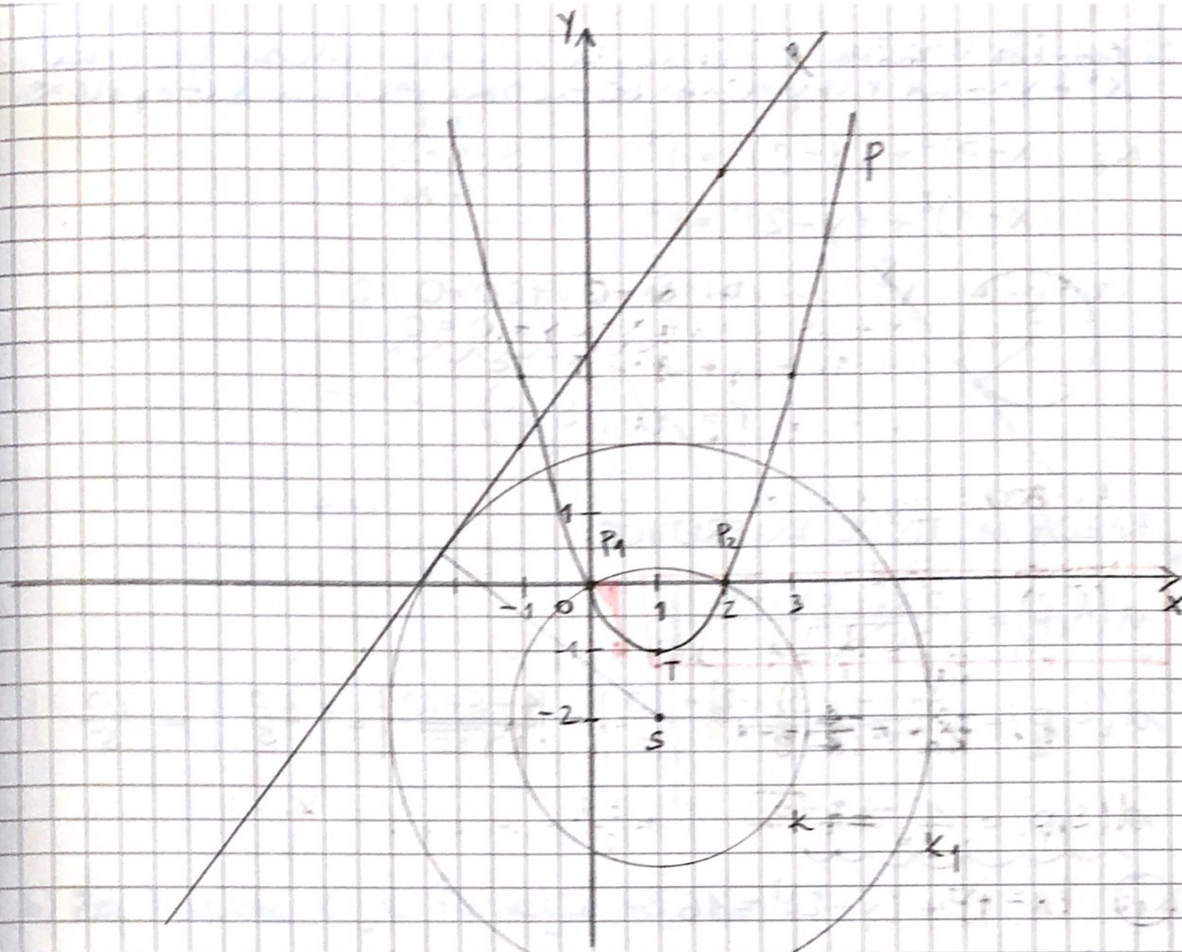
$x_2 = 2$

$x^2 - 2x + 5 = 0$

$D = (-2)^2 - 4 \cdot 1 \cdot 5 = 4 - 20 = -16$ $\quad D < 0$ → ni realnih rešitev

$y_1 = 0$ $\rightarrow P_1(0, 0)$

$y_2 = 4 - 4 = 0$ $\rightarrow P_2(2, 0)$



b) Izračunaj, pod kolikšnim kotom se krivulji sekata!

$tg\,\varphi = \left|\frac{k_2 - k_1}{1 + k_1 \cdot k_2}\right|$

$\rightarrow v\ P_1(0,0)$

$x^2 + y^2 - 2x + 4y = 0$

$kt_1 = y'(x_1)$

<u>odvod:</u> $2x + 2y \cdot y' - 2 + 4y' = 0 \quad /:2$

$y \cdot y' + 2y' = 1 - x$

$y'(y+2) = 1 - x$

$y' = \frac{1-x}{y+2}$, $kt_1 = \frac{1-0}{0+2} = \frac{1}{2}$

$y = x^2 - 2x$

$kt_2 = y'(x_1)$

<u>odvod:</u> $y' = 2x - 2$

$y' = 2 \cdot 0 - 2$

$y' = -2 = kt_2$

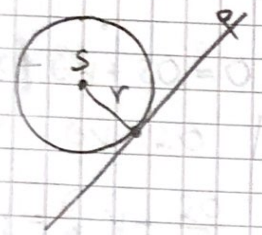
$tg\,\varphi = \left|\frac{-2 - \frac{1}{2}}{1 - 1}\right| = \infty$

$\varphi = 90°$

c) Zapiši enačbo krožnice, ki je koncentrična krožnici $x^2 + y^2 - 2x + 4y = 0$ in se dotika premice $8x - 6y + 20 = 0$.

$K_1: (x-p)^2 + (y-q)^2 = r^2$ $\qquad S(1, -2)$ ($p = x_1$, $q = y_1$)

$(x-1)^2 + (y+2)^2 = r^2$



$p: 8x - 6y + 20 = 0 \;/ :2$

$4x - 3y + 10 = 0$ (a, b, c)

$r = d(S, p)$

$r = 4$

RAZDALJA TOČKE DO PREMICE:

$$d(A, p) = \left| \frac{a x_1 + b y_1 + c}{\sqrt{a^2 + b^2}} \right| \; !$$

$$d(S, p) = \left| \frac{4 \cdot 1 + (-3) \cdot (-2) + 10}{\sqrt{16 + 9}} \right| = \left| \frac{4 + 6 + 10}{\sqrt{25}} \right| = \left| \frac{20}{5} \right| = \frac{20}{5}$$

$d(S, p) = 4 = r$

$K_1: (x-1)^2 + (y+2)^2 = 16$

$x^2 - 2x + 1 + y^2 + 4y + 4 = 16$

$x^2 + y^2 - 2x + 4y - 11 = 0$

1. Izračunaj odvod!

$$y = \frac{\sqrt[3]{x} \cdot \sqrt{x^3}}{\sqrt[4]{x^{-1}} \cdot \sqrt[4]{x^3}} = \frac{x^{\frac{1}{3}} \cdot x^{\frac{3}{6}}}{x^{-\frac{1}{4}} \cdot x^{\frac{3}{4}}} = x^{\frac{1}{3} + \frac{1}{2} + \frac{1}{4} - \frac{3}{4}} = x^{\frac{1}{3} + \frac{1}{2} - \frac{2}{4}}$$

$y = x^{\frac{1}{3}}$

$y' = \frac{1}{3} x^{-\frac{2}{3}} = \frac{1}{3} \sqrt[3]{\frac{1}{x^2}}$

2. $y = \ln(4 - 2x)$

a) Nariši graf funkcije!

$4 - 2x = 0$

$2x = 4$

$x = 2 \rightarrow$ asimptota

zač. vr.: $x = 0$

$y = \ln 2$

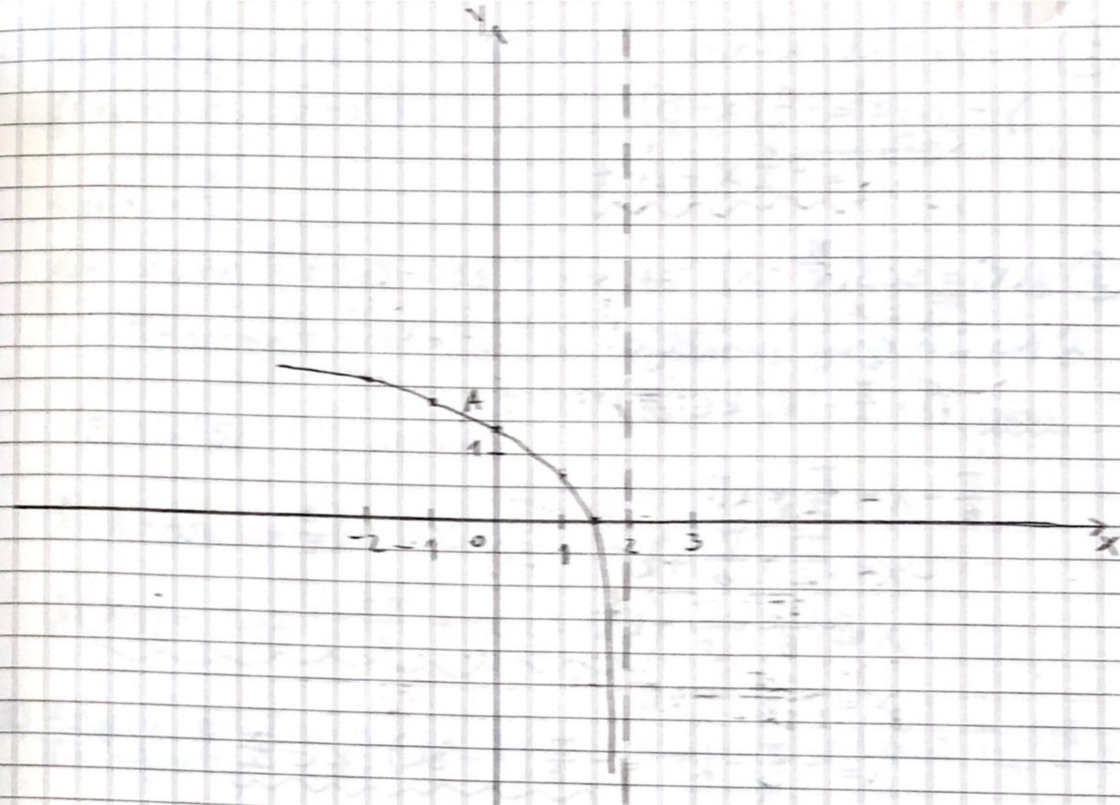
$A(0, \ln 2)$

ničla: $y = 0$ $\quad \ln(4 - 2x) = 0$

$4 - 2x = 1$

$-2x = -3$

$x = \frac{3}{2}$



b) Za katere x je funkcija negativna?

$y < 0$

$\ln(4-2x) < 0$ $\qquad z = 4-2x$

$\ln(z) < 0$

$0 < z < 1$

$0 < 4-2x$ $\qquad 4-2x < 1$

$2x < 4 \quad /:2$ $\qquad -2x < -3 \quad /:(-2)$

$x < 2$ $\qquad x > \frac{3}{2}$

R: $\frac{3}{2} < x < 2$

c) Zapiši enačbo tangente na krivuljo, ki je vzporedna premici $x+2y+6=0$.

$t \parallel p \Leftrightarrow k_t = k_p$

p: $x+2y+6=0$

$2y = -x-6 \quad /:2$

$y = -\frac{1}{2}x - 3 \Rightarrow k_p = -1/2 = k_t$

$k_t = y'(x_1)$

$y' = \frac{1}{4-2x} \cdot (-2) = -\frac{2}{2(2-x)} = -\frac{1}{2-x}$

$-\frac{1}{2-x} = -\frac{1}{2} \quad / \cdot -2(2-x)$ $\qquad 2 = 2-x$ $\qquad y_1 = \ln 4$

$x_1 = 0$

(t): $y - y_1 = k_t(x - x_1)$

$y - \ln 4 = -\frac{1}{2}(x - 0)$

$y = -\frac{1}{2}x + \ln 4$

(3) $f(x) = \cos\left(\frac{\pi}{6} - x\right)$

a) Nariši graf funkcije!

ničle: $\left(x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right)$

$\frac{\pi}{6} - x = \frac{\pi}{2} + k\pi$

$-x = \frac{\pi}{2} - \frac{\pi}{6} + k\pi \quad /\cdot(-1)$

$x = -\frac{3\pi}{6} + \frac{\pi}{6} - k\pi$

$x = -\frac{2\pi}{6} - k\pi$

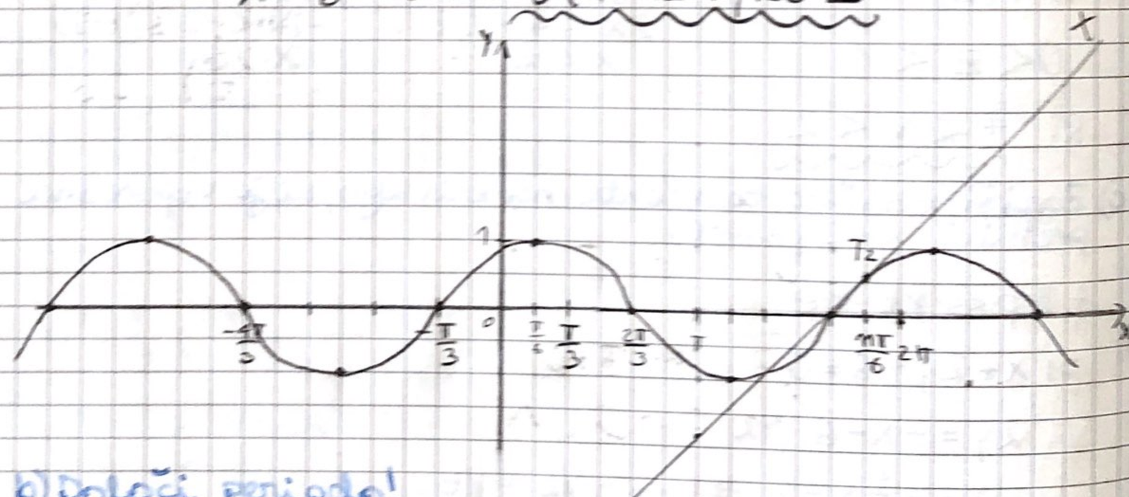
$x = -\frac{\pi}{3} - k\pi = -\frac{\pi}{3}(1 + 3k),\ k \in \mathbb{Z}$

maksimumi: $\left(x = 2k\pi,\ k \in \mathbb{Z}\right)$

$\frac{\pi}{6} - x = 2k\pi$

$-x = -\frac{\pi}{6} + 2k\pi \quad /\cdot(-1)$

$x = \frac{\pi}{6} - 2k\pi = \frac{\pi}{6}(1 - 12k),\ k \in \mathbb{Z}$

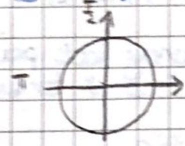


b) Določi periodo!

$\frac{2\pi}{\omega}$ = osn. perioda $\qquad \omega = 1$

osnovna perioda = 2π

c) Izračunaj $f(x_1)$, če je $\sin x_1 = \frac{4}{5}$ in $\frac{\pi}{2} < x_1 < \pi$.



$f(x_1) = \cos\left(\frac{\pi}{6} - x_1\right)$

$\sin x_1 = \frac{4}{5}$

$x_1 = \arcsin\frac{4}{5} + 2k\pi = 53°8' + k\cdot 360°$

$x_2 = \pi - \arcsin\frac{4}{5} + 2k\pi = 126°52' + k\cdot 360°$ $\Big\}\ k \in \mathbb{Z}$

$\frac{\pi}{2} < x_1 < \pi$

$x_1 = 126°52' = \pi - \arcsin\frac{4}{5}$

$f\left(\pi - \arcsin\frac{4}{5}\right) = \cos\left(\frac{\pi}{6} - \pi + \arcsin\frac{4}{5}\right)$

$f(x_1) = \cos\left(-\frac{5\pi}{6} + \arcsin\frac{4}{5}\right) \doteq -0{,}12$

d) Zapiši enačbo tangente na krivuljo v točki $T_2\left(\frac{11\pi}{6}, y_2\right)$! Tangento nariši!

tangenta: $y - y_2 = k_t(x - x_2)$

$k_t = y'(x_2)$

$f'(x) = -\sin\left(\frac{\pi}{6} - x\right)\cdot(-1) = \sin\left(\frac{\pi}{6} - x\right)$

$k_t = \sin\left(\frac{\pi}{6} - \frac{11\pi}{6}\right) = \sin\left(-\frac{10\pi}{6}\right) = \sin\left(-\frac{5\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$y_2 = \cos\left(\frac{\pi}{6} - \frac{11\pi}{6}\right) = \cos\left(-\frac{10\pi}{6}\right) = \cos\left(-\frac{5\pi}{3}\right) = 0'5$

t: $y - \frac{1}{2} = \frac{\sqrt{3}}{2}\left(x - \frac{11\pi}{6}\right)$

$y = \frac{\sqrt{3}}{2}x - \frac{11\sqrt{3}\pi}{12} + \frac{1}{2}$

$y = \frac{\sqrt{3}}{2}x - \frac{11\sqrt{3}\pi + 6}{12}$