Ekstremalni problemi

zgled:

① Število 18 razčleni na dva dela tako, da bo njun produkt največji.

$18 = x + y \Rightarrow y = 18 - x$

P - ekstrem

$P = x \cdot y$

$P(x) = x \cdot (18 - x) = 18x - x^2$

$P'(x) = 18 - 2x = 0$

$x = 9 \quad \Rightarrow y = 9$

Dokaz: $P''(x) = -2$

$P''(x) < 0$

$\Downarrow$

maksimum

O: Produkt bo največji, ko bomo število razčlenili na dva enaka dela (9 in 9).

② Kolikšne morajo biti dimenzije valja, da bomo za V=1000L porabili najmanj materiala?

valj

$V = 1000\,L$

$1L = 1\,dm^3$

$V = \pi r^2 \cdot v$

$\pi r^2 \cdot v = 1000$

$v = 1000/\pi r^2$

P - minimalna

$P = 2S_O + S_{pl}$

$P = 2\pi r^2 + 2\pi r \cdot v$

$P(r) = 2\pi r^2 + 2\pi r \cdot \left(\frac{1000}{\pi r^2}\right)$

$P(r) = 2\pi r^2 + 2000\, r^{-1}$

$P'(r) = 4\pi r - 2000\, r^{-2} = 0$

$4\pi r - 2000\, r^{-2} = 0 \quad /:4$

$\pi r - 500\, r^{-2} = 0$

$r^{-2}(\pi r^3 - 500) = 0 \quad /: r^{-2} \neq 0$

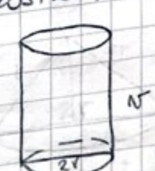
$\pi r^3 - 500 = 0$

$r^3 = \frac{500}{\pi}$

$r = \sqrt[3]{\frac{500}{\pi}} = 5{,}42\ dm$

$v = \frac{1000}{\pi \cdot 5{,}42^2} = 10{,}84\ dm$

ENAKOSTRANIČNI VALJ



$2r = v$

Dokaz:

$P''(r) = 4\pi + 4000$

$P''(r) > 0 \Rightarrow$ minimum

③ Kolikšne morajo biti dimenzije valja, da bo volumen največji? P je podana.

V – maksimum

$$P = 2\pi r^2 + 2\pi r v$$

$$V = \pi r^2 \cdot v$$

$$2\pi r v = P - 2\pi r^2 \quad /: 2\pi r$$

$$v = \frac{P - 2\pi r^2}{2\pi r}$$

$$V_{(r)} = \cancel{\pi} r^{\cancel{2}} \cdot \frac{P - 2\pi r^2}{2\cancel{\pi}\cancel{r}}$$

$$V_{(r)} = \frac{P \cdot r - 2\pi r^3}{2} = \frac{1}{2} P \cdot r - \pi r^3$$

$$V_{(r)}' = \frac{P}{2} - 3\pi r^2 = 0$$

$$3\pi r^2 = \frac{P}{2}$$

$$r^2 = \frac{P}{6\pi}$$

$$r = \sqrt{\frac{P}{6\pi}}$$

$$v = \frac{P - \cancel{2}^{1}\cancel{\pi}\frac{P}{_{2}\cancel{6}\cancel{\pi}}}{2\pi\sqrt{\frac{P}{6\pi}}} = \frac{P - \frac{P}{3}}{2\pi\sqrt{\frac{P}{6\pi}}} = \frac{\frac{\cancel{2}}{3}P}{\cancel{2}\pi\sqrt{\frac{P}{6\pi}}} = \frac{P\sqrt{\frac{P}{6\pi}}}{3\pi\sqrt{\frac{P}{6\pi}}\cdot\sqrt{\frac{P}{6\pi}}}$$

$$v = \frac{\cancel{P}\sqrt{\frac{P}{6\pi}}}{\cancel{3\pi}\frac{\cancel{P}}{_{2}\cancel{6\pi}}} = 2\sqrt{\frac{P}{6\pi}}$$

Valj je enakostraničen.

$2r = v$

DIMENZIJE: r, $v = 2r$

4. Izračunaj razsežnosti odkritega bazena s kvadratnim dnom in $V = 256\ m^3$ tako, da bo za betoniranje sten in dna treba najmanj materiala.

$V = 256\ m^3$

P - minimum

$P = S_o + S_{pl}$

$P = a^2 + 4a \cdot v$

$P(a) = a^2 + 4\cancel{a} \cdot \frac{256}{a^{\cancel{2}}}$

$P(a) = a^2 + 1024 a^{-1}$

$V = S_o \cdot v$

$V = a^2 \cdot v = 256\ m^3$

$v = \frac{256}{a^2}$

$P'(a) = 2a - 1024 a^{-2} = 0$

$2a - 1024 a^{-2} = 0 \quad / :2$

$a - 512 a^{-2} = 0$

$a^{-2}(a^3 - 512) = 0 \quad / : a^{-2} \neq 0$

$a^3 - 512 = 0$

$a^3 = 512$

$a = 8\ m \Rightarrow v = \frac{256}{64} = 4\ m$

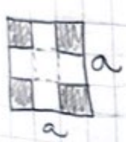
DIMENZIJE $(8 \times 8 \times 4)\ m^3$

Dokaz:

$P''(a) = 2 + 2048\ a^{-3}$

$P''(8) = 2 + 2048 \cdot 8^{-3} = 6 \qquad P''(8) > 0 \Rightarrow$ minimum

5. Ob ogliščih kvadratne plošče z robom a izreži take skladne kvadrate, da boš naredil škatlo brez pokrova, ki bo imela največjo prostornino.



V - maksimum

$V = a \cdot b \cdot c$

$V = (a - 2x)^2 \cdot x$

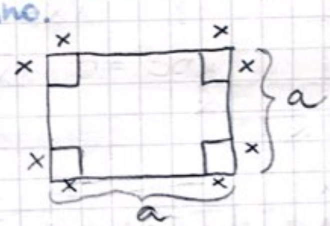
$V = (a^2 - 4ax + 4x^2)x$

$V = a^2x - 4ax^2 + 4x^3$

$V' = a^2 - 8ax + a^2 = 0$

$(6x - a)(2x - a) = 0$

1.) $x = \frac{a}{6}$ max 2.) $x = \frac{a}{2}$ min



DOKAZ:

$V' = -8a = -8\frac{x}{6} < 0$ max