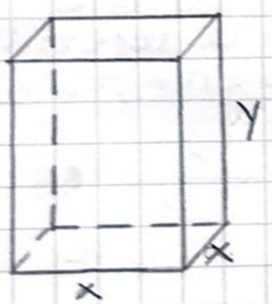
$x^2 = -ab$ R: $x = \sqrt{ab}$

list

A3 3m žice

OS. PLOSKEV = kvadrat

V - maksimum



$V = a \cdot b \cdot c$

$V = x^2 \cdot y$

$8x + 4y = 3\,m$

$y = \frac{3}{4} - 2x$

$x^2 + ab - 2x^2 = 0$

$x^2 = ab$

$x = \sqrt{ab}$ geometrijska sredina

f' _ mat. dokaz!

$V(x) = x^2 \cdot \left(\frac{3}{4} - 2x\right) = \frac{3}{4}x^2 - 2x^3$

$V'(x) = 0$

$V'(x) = \frac{3}{2}x - 6x^2 = 0$

$\frac{3}{2}x - 6x^2 = 0 \quad / \cdot 2$

$3x - 12x^2 = 0 \quad / : 3$

$x - 4x^2 = 0 \quad / \cdot (-1)$

$4x^2 - x = 0$

$x(4x - 1) = 0$

$x_1 = 0, \; x_2 = \frac{1}{4}$

$x = \frac{1}{4} m$, $y = \frac{3}{4} - \cancel{2}^{1} \cdot \frac{1}{\cancel{4}} = \frac{1}{4} m$

$y'' = \frac{3}{2} - 12x$

$y''\left(\frac{1}{4}\right) = \frac{3}{2} - \cancel{12}^{6} \cdot \frac{1}{\cancel{4}_2} = \frac{3}{2} - \frac{6}{2} = -\frac{3}{2}$

$y''\left(\frac{1}{4}\right) < 0 \Rightarrow$ maksimum

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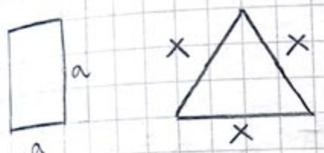
12m



$S_{\Delta + \square}$ - minimalna

$S = a^2 + \frac{x^2\sqrt{3}}{4}$

$12 = 4a + 3x$

$4a = 12 - 3x$

$a = 3 - \frac{3}{4}x$

$S = \left(3 - \frac{3}{4}x\right)^2 + \frac{x^2\sqrt{3}}{4}$

$S = 9 - \frac{9}{2}x + \frac{9}{16}x^2 + \frac{x^2\sqrt{3}}{4}$

$S' = 0$

$S' = -\frac{9}{2} + \frac{9}{8}x + \frac{\sqrt{3}}{2}x = 0$

$-9 + \frac{9}{4}x + \sqrt{3}x = 0$

$\frac{9 + 4\sqrt{3}}{4} x = 9 \quad / \cdot 4$

$(9 + 4\sqrt{3})x = 36$

$x = \frac{36}{9 + 4\sqrt{3}} = 2{,}26$

→ $f(x) = x^2 \cdot \ln x$

Določi največjo in najmanjšo vrednost funkcije na intervalu $(0, 2]$. (globalni ekstrem)

$f'(x) = 0$

$f'(x) = 2x \cdot \ln x + x^{\cancel{2}} \cdot \frac{1}{\cancel{x}} = 2x \cdot \ln x + x$

$2x \cdot \ln x + x = 0$

$x(2\ln x + 1) = 0$

1.) $x = 0$ // ker naravni log. ne obstaja

2.) $2\ln x + 1 = 0$

$\ln x = -\frac{1}{2}$

$e^{-\frac{1}{2}} = x$

$y = \left(e^{-\frac{1}{2}}\right)^2 \cdot \ln\left(e^{-\frac{1}{2}}\right)$

$y = e^{-1} \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2}e^{-1}$

$T(e^{-\frac{1}{2}}, -\frac{1}{2}e^{-1})$

$f''(x) = 2\cdot \ln x + \cancel{2x}\cdot\frac{1}{\cancel{x}} + 1$

$f''(x) = 2\ln x + 3$

$f''(e^{-\frac{1}{2}}) = 2\ln(e^{-\frac{1}{2}}) + 3$

$f''(e^{-\frac{1}{2}}) = \cancel{2}\cdot(-\frac{1}{\cancel{2}}) + 3 = -1+3 = 2$

$f''(e^{-\frac{1}{2}}) > 0 \Rightarrow$ lokalni minimum

V KRAJIŠČIH:

$f(0{,}1) = -0{,}023$ $\qquad$ $f(2) = 4\cdot\ln 2 \doteq 2{,}77$

$\Downarrow$ $T(e^{-\frac{1}{2}}, -\frac{1}{2}e^{-1}) \sim$ globalni minimum

$A(2, 4\cdot\ln 2) \sim$ globalni maksimum

→ Na elipsi $2x^2 + y^2 = 18$ sta dani točki $T_1(1, y_1 > 0)$, $T_2(3, y_2)$. Določi na elipsi T_3 tako, da bo ploščina $\Delta T_1T_2T_3$ največja. Kolikšna je!

$y_1 = \pm\sqrt{18-2x^2}$

$y_1 = \pm\sqrt{18-2\cdot 1} = \pm 4 \qquad y_1 > 0$

$T_1(1, 4)$ $\qquad$ $S = 3\sqrt{6}+6$

$-\sqrt{6}$

$-\sqrt{6}$

$y_2 = \pm\sqrt{18-2\cdot 3^2}$

$y_2 = \pm\sqrt{0} = 0 \qquad T_2(3, 0)$

$S\Delta = \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$S\Delta = \frac{1}{2}[1(0-y_3) + 3(y_3-4) + x_3(4-0)]$

$S\Delta = \frac{1}{2}[-y_3 + 3y_3 - 12 + 4x_3]$

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$S\Delta = \frac{1}{2}[2y_3 + 4x_3 - 12]$

$S\Delta = y_3 + 2x_3 - 6$

$2(x_3)^2 + (y_3)^2 = 18$

$(y_3)^2 = 18 - 2(x_3)^2$

$y_3 = \pm\sqrt{18-2(x_3)^2}$

$S(y_3) = \sqrt{18-2(x_3)^2} + 2x_3 - 6$

$$S'(x_3) = \frac{1}{\cancel{2}} \cdot \frac{1}{\sqrt{18-2(x_3)^2}} \cdot (-\cancel{4}^{2} x_3)$$

$$S'(x_3) = \frac{-x_3 \cdot \sqrt{18-2(x_3)^2}}{\sqrt{18-2(x_3)^2} \cdot \sqrt{18-2(x_3)^2}} = \frac{-x_3 \sqrt{18-2(x_3)^2}}{18-2(x_3)^2}$$

$$-x_3 \sqrt{18-2(x_3)^2} = 0 \quad /^2$$

$$(x_3)^2 \cdot (18-2(x_3)^2) = 0$$

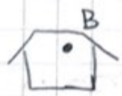
$$18(x_3)^2 - 2(x_3)^4 = 0 \quad /:2$$

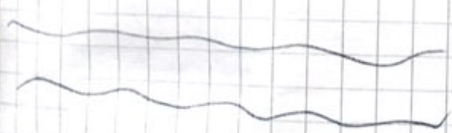
$$\underbrace{(x_3)^2}_{\to 0} (9-(x_3)^2) = 0$$

$$9-(x_3)^2 = 0$$

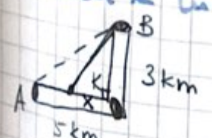
$$(3-x)(3+x) = 0$$

$$x_1 = 3, \quad x_2 = -3$$





→ Med krajema A in B, povezanima s cesto od kraja A do križišča K in od K do B, je treba položiti plinsko napeljavo. 1m napeljave ob cesti stane 50 ~~000 SIT~~ €. 1m napeljave po terenu pa 70 ~~000 SIT~~ €. Kako je treba napeljati, da bo najceneje?



8km OB CESTI

(ekstrem)

(   )

PAZI!

m — km

OPTIMALNA POT:

C - najmanjša

$$C = (5-x) \cdot 50\,000 + \sqrt{x^2+9} \cdot 70\,000$$

$$C = 250\,000 - 50\,000x + (x^2+9)^{\frac{1}{2}} \cdot 70\,000$$

$$C' = -50\,000 + \frac{1}{\cancel{2}}(x^2+9)^{-\frac{1}{2}} \cdot (\cancel{2}x) \cdot 70\,000 = 0$$

$$5\cancel{0}\,\cancel{000} = 7\cancel{0}\,\cancel{000}\,(x^2+9)^{-\frac{1}{2}} \cdot x$$

$$\frac{x}{\sqrt{x^2+9}} = \frac{5}{7} \qquad 7x = 5\sqrt{x^2+9} \quad /^2$$

$$49x^2 = 25(x^2+9)$$

$$24x^2 = 225$$

$$x = \sqrt{9{,}375} \doteq 3{,}06$$