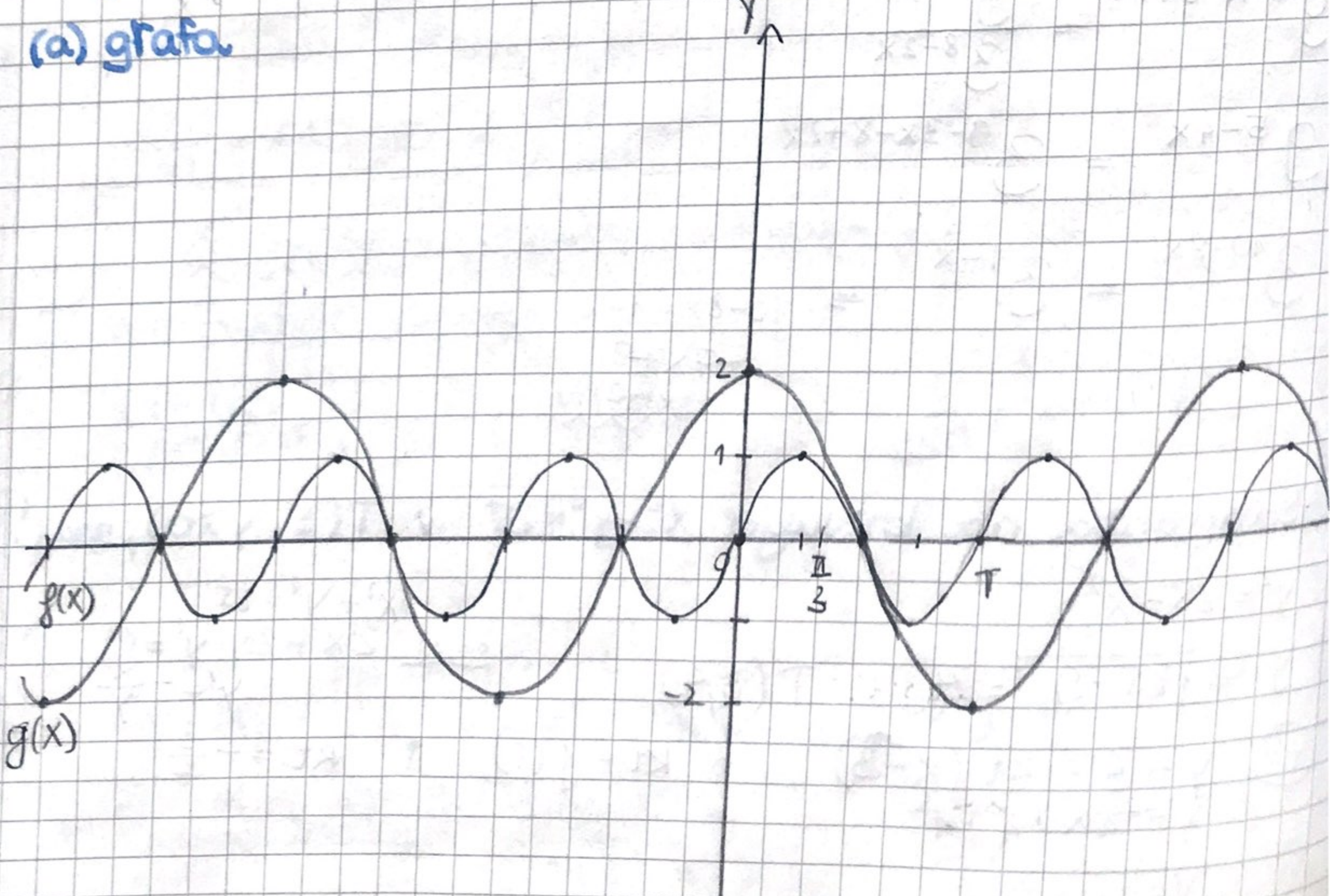
II. del : STRUKTURIRANE NALOGE

1. $f(x) = \sin 2x$

$g(x) = 2\cos x$

(a) grafa



(b) $f(x) = g(x)$

$\sin 2x = 2\cos x$

$2\cdot \sin x \cdot \cos x - 2\cdot \cos x = 0 \quad /:2$

$\sin x \cdot \cos x - \cos x = 0$

$\cos x(\sin x - 1) = 0$

1.) $\cos x = 0$ 2.) $\sin x = 1$

$x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$ $x = \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}$

R: $x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

(c) $h(x) = 3f(x) - 2g(x)$ odvod, nedol. integral = ?

$h(x) = 3\cdot \sin 2x - 4\cos x$

$h'(x) = 3\cdot 2\cos 2x + 4\sin x$

$h'(x) = 6\cos 2x + 4\sin x$

$\int (3\sin 2x - 4\cos x)\,dx = 3\int \sin 2x\,dx - 4\int \cos x\,dx =$

$= -3\cdot \frac{\cos 2x}{2} - 4\cdot(\sin x) + C = -\frac{3}{2}\cos 2x - 4\sin x + C$

② parabola: $y = x^2 - 4x + 5$
premica: $y = -x + 5$

(a) grafa:

$y = x^2 - 4x + 5$

$y' = 2x - 4 = 0$

$2x = 4$

$x = 2$, $y = 4 - 8 + 5 = 1$

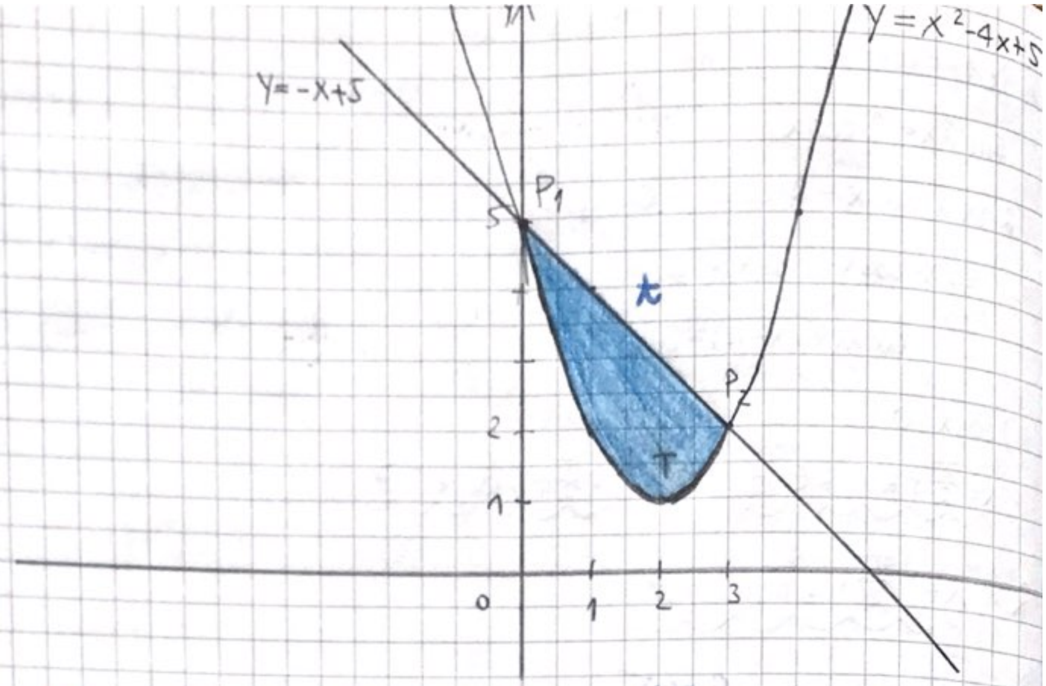
Teme: $T(2,1)$

zač. vr.: $A(0,5)$

ničle: $y = x^2 - 4x + 5$

$D = 16 - 20 = -4$

$D < 0$, ni R ničel



(b) dolžina tetive

PRESEČIŠČE:

$y = x^2-4x+5$, $y = -x+5$

$x^2 - 4x + \cancel{5} + x - \cancel{5} = 0$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x_1 = 0$, $x_2 = 3$ $\quad P_1(0,5)$

$y_1 = 5$, $y_2 = 2$ $\quad P_2(3,2)$

$t = d(P_1, P_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$t = \sqrt{(3-0)^2 + (2-5)^2} = \sqrt{9+9} = 3\sqrt{2}$

(c) ploščina lika, ki ga oklepata krivulji

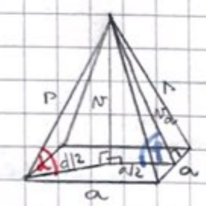
$$S = \int_0^3 \left(-x + \cancel{5} - x^2 + 4x - \cancel{5}\right) dx = \int_0^3 \left(-x^2 + 3x\right) dx =$$

$$= \left(-\frac{x^3}{3} + \frac{3x^2}{2}\right)\Big|_0^3 = -\frac{27}{3} + \frac{27}{2} = \frac{-18+27}{2} = \frac{9}{2} e^2$$

③ Pravilna 4-strana piramida:

$P = 360 \, cm^2$

$v_a = 13 \, cm$



(a) skica, V

$P = S_O + S_{pl} = a^2 + \cancel{4}^2 \cdot \frac{a \cdot v_a}{\cancel{2}} = a^2 + 2a \cdot v_a$

$360 = a^2 + a \cdot 26$

$a^2 + 26a - 360 = 0$

$(a-10)(a+36) = 0$

$a_1 = 10$, $a_2 = -36$ // dol. str. ne more biti neg. št.

$a = 10 \, cm$

$v^2 = v_a^2 - \left(\frac{a}{2}\right)^2$

$v^2 = (13\,cm)^2 - (5\,cm)^2 = 144\,cm$

$v = 12\,cm$

$$V = \frac{S_O \cdot v}{3} = \frac{a^2 \cdot v}{3} = \frac{100\,cm^2 \cdot \cancel{12}^{4}\,cm}{\cancel{3}_1} = \underline{400\,cm^3}$$

(b) kot med stransko in osnovno ploskvijo

$$\sin \varphi = \frac{v}{v_a} = \frac{12\,\cancel{cm}}{13\,\cancel{cm}} \Rightarrow \varphi = 67{,}4^\circ = \underline{67^\circ 23'}$$

(c) kot med stranskim robom in osnovno ploskvijo

$$d = a\sqrt{2} = 10\sqrt{2}\,cm$$

$$\operatorname{tg} \alpha = \frac{v}{d/2} = \frac{12\,\cancel{cm}}{5\sqrt{2}\,\cancel{cm}} \Rightarrow \underline{\alpha = 59^\circ 29'}$$

4. $p(x) = x^3 - 3x$

(a) ničle, graf

$$x^3 - 3x = 0$$
$$x(x^2-3) = 0$$
$$x(x-\sqrt{3})(x+\sqrt{3}) = 0$$

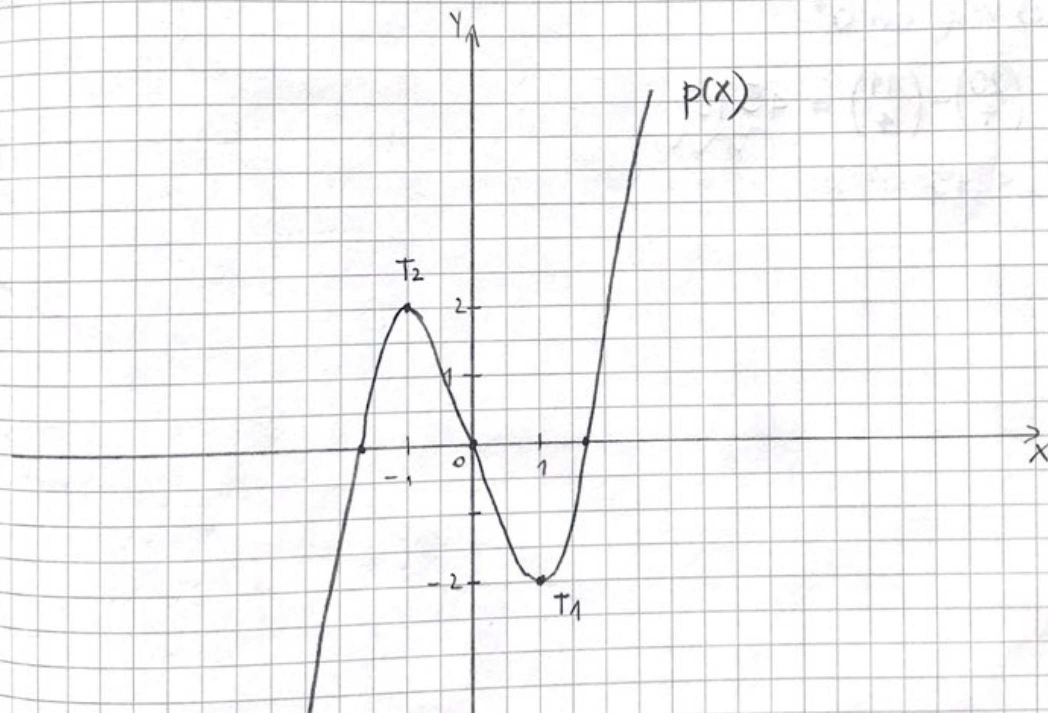
$$\underline{x_1 = 0,\ x_2 = \sqrt{3},\ x_3 = -\sqrt{3}}$$

$$y' = 3x^2 - 3 = 0$$
$$x^2 - 1 = 0$$
$$(x-1)(x+1) = 0$$
$$x_1 = 1,\ x_2 = -1$$
$$\underline{y_1 = -2,\ y_2 = 2}$$

$$y'' = 6x$$

$T_1(1,-2)$ mini

$T_2(-1,2)$ max



(b) $p(x) = 2$

$$x^3 - 3x = 2$$
$$x^3 - 3x - 2 = 0$$

	1	0	-3	-2
		-1	1	2
-1	1	-1	-2	$\boxed{0}$

$\underline{x_1 = -1}$

$$x^2 - x - 2 = 0$$
$$(x-2)(x+1) = 0$$
$$\underline{x_2 = 2,\ x_3 = -1}$$

R: $\underline{x_1 = -1,\ x_2 = 2}$

(c) Pokaži, da je polinom p liha funkcija.

$p(x) = x^3 - 3x$ $\qquad -p(x) = -x^3 + 3x$

$p(-x) = (-x)^3 - 3(-x)$

$p(-x) = -x^3 + 3x$

liha funk. $\quad p(-x) = -p(x)$

$-x^3 + 3x \overset{?}{=} -x^3 + 3x$ ✓

5. 9 dečkov
11 deklic

sestavimo 4-člansko skup.

(a) v njej ni nobene deklice

$\binom{9}{4} = 126$

(b) 2× ♂ & 2× ♀

$C_9^2 \quad C_{11}^2$

$\binom{9}{2} \cdot \binom{11}{2} = 36 \cdot 55 = 1980$

(c) vsaj en ♂

$\binom{20}{4} - \binom{11}{4} = 4515$