8

① $D_f = \mathbb{R}$

$Z_f = (-\infty, 0]$

Funkcija je soda, ker je graf zrcalna preko y osi.

$f(-1) = -2$

$f'(-1) = 0$

$k_t = 0$

$x = -1$ (enačba normale)

predznak $f'(-\frac{1}{2})$: +

predznak $f'(\frac{1}{2})$: −

12/15

② $f(x) = \dfrac{x^3 \cdot \sqrt{x}}{\sqrt[3]{x \cdot \sqrt[4]{x}}}$

$f(x) = \dfrac{x^3 \cdot x^{\frac{1}{2}}}{\sqrt[3]{x \cdot x^{\frac{1}{4}}}}$

$\left(x^{\frac{5}{4}}\right)^{\frac{1}{3}} = x^{\frac{5}{12}}$

$f(x) = \dfrac{x^{\frac{7}{2}}}{\sqrt[3]{x^{\frac{5}{4}}}}$

$f(x) = \dfrac{x^{\frac{7}{2}}}{x^{\frac{5}{12}}}$

$f(x) = x^{\frac{37}{12}}$

$f'(x) = \frac{37}{12} x^{\frac{25}{12}}$

po pravilu produkta

odvod je enak 0

$g(x) = \sin x \cdot x^2 + \pi x^1 + \pi$

$g'(x) = \cos x \cdot x^2 + \sin x \cdot 2x + \pi$

$\ln x \xrightarrow{\text{odvod}} \frac{1}{x}$

$h(x) = \ln(1 - x^{-1})$ Z

$h'(x) = \dfrac{1}{1 - \frac{1}{x}} \cdot Z'$

$h'(x) = \dfrac{1}{\frac{x-1}{x}}$

$h'(x) = \dfrac{x}{x-1}$

③ $f(x) = \dfrac{6x}{x^2 - 5x + 4}$

Začetna vrednost: POGOJ: $x = 0$

$f(0) = \frac{0}{4} = 0$

Potek f. pri velikih x:

$6x : (x^2 - 5x + 4) = 0 + k(x)$

Vodoravna asimptota: $y = 0$

Ničle: POGOJ: števec = 0

$6x = 0 \; |:6$

$x = 0$ (1.st.)

Poli: POGOJ: imen. = 0

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x_1 = 4$ (1.st.) $x_2 = 1$ (1.st.)

Stacionarne točke: POGOJ: $f'(x) = 0$

$f'(x) = \dfrac{(6x)' \cdot (x^2-5x+4) - 6x \cdot (x^2-5x+4)'}{(x^2-5x+4)^2}$

$f'(x) = \dfrac{6 \cdot (x^2-5x+4) - 6x \cdot (2x-5)}{(x^2-5x+4)^2}$

$f'(x) = \dfrac{6x^2 - 30x + 24 - 12x^2 + 30x}{(x^2-5x+4)^2}$

$f'(x) = \dfrac{-6x^2 + 24}{(x^2-5x+4)^2}$

$\dfrac{-6x^2+24}{(x^2-5x+4)^2} = 0 \; |\cdot (x^2-5x+4)^2$

$-6x^2 + 24 = 0 \; |:(-6)$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$x_1 = -2 \quad x_2 = 2$

$y_1 = -\frac{2}{3} \quad y_2 = -6$

$T_1(-2, -\frac{2}{3})$

$T_2(2, -6)$

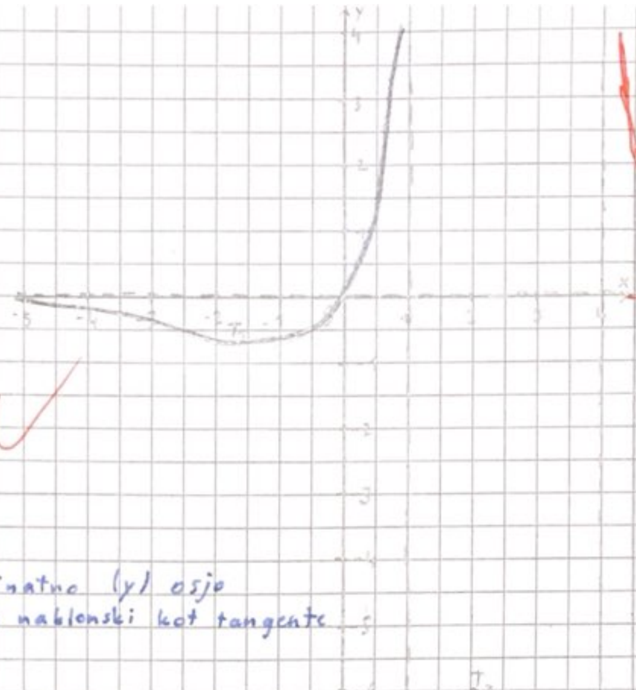
Določitev ekstrema:

x	-3	-2	-1
y'	$+$	0	$-$

$T_1(-2, -\frac{1}{3})$ je lokalni maksimum

x	1	2	3
y'	$-$	0	$+$

$T_2(2,-6)$ je lokalni minimum



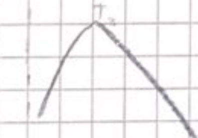
23/24

Kot med funkcijo in ordinatno (y) osjo bo enak $\varphi = 90° - \alpha$ → naklonski kot tangente

Odvod funkcije:

$$f'(x) = \frac{-6x^2 + 24}{(x^2-5x+4)^2}$$

Začetna vrednost: $y = 0 \Rightarrow T_3(0,0)$



$$f'(0) = \frac{24}{16} = \frac{3}{2} = k_t = \tan\alpha$$

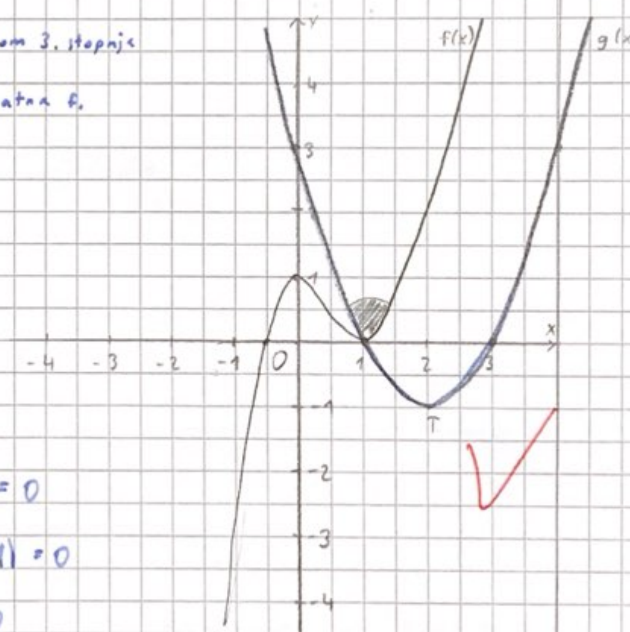
$$\tan\alpha = \frac{3}{2}$$

$$\alpha = 56°19'$$

$$\varphi = 90° - 56°19' = 33°41'$$

4. $f(x) = 2x^3 - 3x^2 + 1$ → polinom 3. stopnje

$g(x) = x^2 - 4x + 3$ → kvadratna f.



a) Presečišče:

$$y = y$$

$$2x^3 - 3x^2 + 1 = x^2 - 4x + 3$$

$$2x^3 - 4x^2 + 4x - 2 = 0 \quad /:2$$

$$x^3 - 2x^2 + 2x - 1 = 0$$

$$(x^3 - 1) - 2x(x-1) = 0$$

$$(x-1)(x^2 + x + 1) - 2x(x-1) = 0$$

$$(x-1)(x^2 + x + 1 - 2x) = 0$$

$$x_1 = 1$$

$$y_1 = 0$$

$$P(1,0)$$

$g(x)$: Začetna vrednost: $x = 0$

$f(0) = 3$

Ničle: $f(x) = 0$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x_1 = 3 \quad x_2 = 1$$

Teme: $T(p,q) \Rightarrow T(2,-1)$

$$p = \frac{-b}{2a} = \frac{4}{2} = 2$$

$$q = \frac{-D}{4a} = \frac{-4}{4} = -1$$

$$D = b^2 - 4ac = 4$$

$f(x)$: Ničle: $f(x)=0$

$2x^3-3x^2+1=0$

kandidati: $\pm 1, \pm \frac{1}{2}$

	2	-3	0	1
		2	-1	-1
1	2	-1	-1	0

$\boxed{x_{1,2} = \underline{1} \ (2.st.)}$

$2x^2-x-1=0$

$(2x+1)(x-1)=0$

$x_2 = 1$

$2x = -1 / :2$

$\boxed{x_3 = -\frac{1}{2} \ (1.st.)}$

Začetna vrednost: $x=0$

$f(0) = \underline{1}$

Potek f. pri velikih x:

$x \to +\infty, \ f(x) \to +\infty$

$x \to -\infty, \ f(x) \to -\infty$

Stacionarne točke: pogoj: $f'(x)=0$

$f'(x) = 6x^2-6x$

$6x^2-6x=0 / :6$

$x^2-x=0$

$x(x-1)=0$

$x_1 = \underline{0} \quad x_2 = \underline{1}$

$y_1 = \underline{1}$

$y_2 = \underline{0}$

$\underline{P_1(0,1)} \quad \underline{P_2(1,0)}$

b) Kot med krivuljama:

$\tan \varphi = \left| \frac{k_2-k_1}{1+k_2 \cdot k_1} \right|$

$\tan \varphi = \left| \frac{-2-0}{1-2\cdot 0} \right|$

$\tan \varphi = 2$

$\varphi = \underline{63{,}43^\circ}$

Kot med krivuljama bo enak ~~kotu~~ kotu med tangentama, ki potekata skozi presečišče. $P(1,0)$

Odvod $f(x)$:

~~[illegible]~~ $f'(x) = x^2-x$

$f'(1) = \underline{0} = k_{t_1}$

Odvod $g(x)$:

$g'(x) = 2x-4$

$g'(1) = \underline{-2} = k_{t_2}$

c) Globalni ekstremi na intervalu $[0,3]$:

Globalni ekstremi so ali v lokalnih ekstremih ali pa v ~~[illegible]~~ krajiščih intervala.

$f(0) = \underline{1} \to$ ni globalni minimum

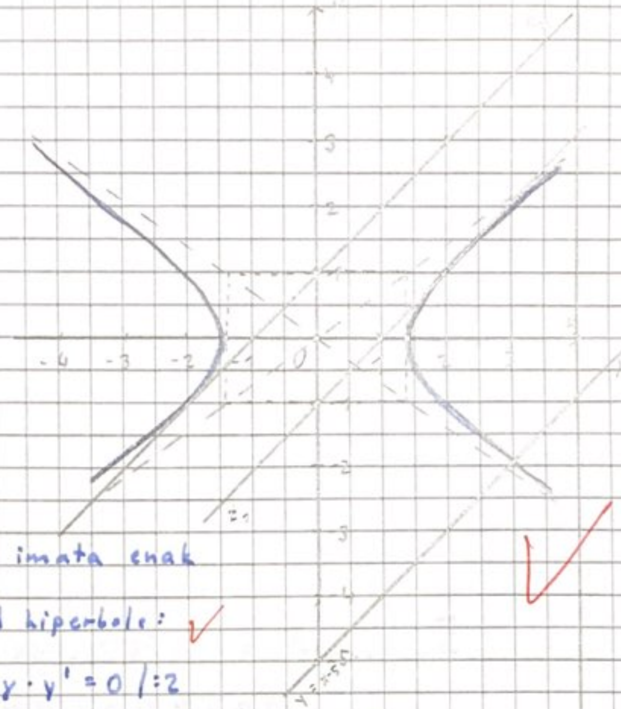
$f(3) = \underline{28} \to$ globalni maksimum: $\underline{G_{max}(3,28)}$

Globalni minimum: $\underline{G_{min}(1,0)}$

(5.) $x^2 - 2y^2 = 2$

vzp. premici $x - y = 5$

enačba tangente, graf



H: $x^2 - 2y^2 = 2 \,/:2$

$$\frac{x^2}{2} - \frac{y^2}{1} = 1$$

$S(0,0)$ $a = \sqrt{2}$, $b = 1$

$y = x - 5$

Ker sta premici vzporedni, imata enak koeficient.

$$k_p = \underset{y'(x)}{\underset{\parallel}{k_t}} = 1$$

Odvod hiperbole:

$$2x - 4y \cdot y' = 0 \,/:2$$
$$x - 2y \cdot y' = 0$$
$$-2y \cdot y' = -x \,/:(-2y)$$
$$y' = \frac{x}{2y}$$
$$\frac{x}{2y} = 1 \,/\cdot 2y$$
$$x = 2y$$

Tangenta:

$$y = kx + n$$
$$y = x + n$$

Dotikališče:

$$x^2 - 2y^2 = 2$$
$$x^2 - 2\cdot(x+n)^2 = 2$$
$$x^2 - 2\cdot(x^2 + 2xn + n^2) = 2$$
$$x^2 - 2x^2 - 4xn - 2n^2 - 2 = 0$$
$$-x^2 - 4xn - 2n^2 - 2 = 0$$
$$D = 0$$
$$b^2 - 4ac = 0$$
$$(-4n)^2 - 4\cdot(-1)\cdot(-2n^2 - 2) = 0$$
$$16n^2 + 4(-2n^2 - 2) = 0$$
$$16n^2 - 8n^2 - 8 = 0 \,/:8$$
$$2n^2 - n^2 - 1 = 0$$
$$n^2 - 1 = 0$$
$$(n+1)(n-1) = 0$$
$$n_1 = -1 \qquad n_2 = 1$$

dobili bomo 2 tangenti

Tangenta 1:

$y_1 = x - 1$

Tangenta 2:

$y_2 = x + 1$

6.) $f(x) = \sqrt{1-2x} - 3$

enačba tangente v ničli

Ničle: pogoj: $f(x) = 0$

$\sqrt{1-2x} - 3 = 0$

$\sqrt{1-2x} = 3 \quad /^2$

$1 - 2x = 9$

$-2x = 8 \quad /:(-2)$

$x = -4$

$T(\overset{x_1}{-4}, \overset{y_1}{0})$

Odvod funkcije:

$f(x) = (1-2x)^{\frac{1}{2}} - 3$

$f'(x) = \frac{1}{2} \cdot (1-2x)^{-\frac{1}{2}} \cdot (-2) - 0$

$f'(x) = -(1-2x)^{-\frac{1}{2}}$

$f'(-4) = -(1+8)^{-\frac{1}{2}}$

$f'(-4) = -(9)^{-\frac{1}{2}}$

$f'(-4) = -\left(\frac{1}{9}\right)^{\frac{1}{2}}$

$f'(-4) = -\frac{1}{3}$

Enačba tangente:

$y - y_1 = f'(x_1)(x - x_1)$

$y - 0 = -\frac{1}{3} \cdot (x + 4)$

$y = -\frac{1}{3}x - \frac{4}{3}$

12

7.) $f(x) = 2\cos x - 3\tan x$

$(\tan x)' = \frac{1}{\cos^2 x}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos\frac{\pi}{3} = \frac{1}{2}$

a) odvod v točki $x = \frac{\pi}{3}$

$f'(x) = -2\sin x - 3 \cdot \left(\frac{1}{\cos^2 x}\right)$

$f'\left(\frac{\pi}{3}\right) = -2 \cdot \sin\frac{\pi}{3} - 3 \cdot \frac{1}{\cos^2\frac{\pi}{3}}$

$f'\left(\frac{\pi}{3}\right) = -\cancel{2}^{1} \cdot \frac{\sqrt{3}}{\cancel{2}_{1}} - 3 \cdot \frac{1}{\frac{1}{4}}$

$f'\left(\frac{\pi}{3}\right) = -\sqrt{3} - \frac{\frac{3}{1}}{\frac{1}{4}}$

$f'\left(\frac{\pi}{3}\right) = -\sqrt{3} - \frac{4}{3}$ 12

$f'\left(\frac{\pi}{3}\right) = \frac{-4-3\sqrt{3}}{3}$

b) Enačba normale v točki z absciso $x = 0$

$k_t = y'$

Enačba normale:

$\boxed{k_N = -\frac{1}{k_t}}$

$f'(x) = -2\sin x - 3 \cdot \left(\frac{1}{\cos^2 x}\right)$

$f'(0) = -2 \cdot 0 - 3 \cdot \frac{1}{1}$

$f'(0) = -3 = k_t$

$k_N = -\frac{1}{-3} = \frac{1}{3}$

$T(0, y_1) \Rightarrow T(\overset{x_1}{0}, \overset{y_1}{2})$

$y = 2 \cdot \cos 0 - 3 \cdot \tan 0$

$y = 2 \cdot 1 - 3 \cdot 0$

$y = 2$

$y - y_1 = k_N \cdot (x - x_1)$

$y - 2 = \frac{1}{3}(x - 0)$

$y = \frac{1}{3}x + 2$

c) naraščanje / padanje v točki z absciso $x=\frac{\pi}{4}$

Če bo odvod v tej točki pozitiven, bo naraščala, če bo negativen, bo padala, če bo 0, pa bo stagnirala.

$f'(x) = -2\sin x - 3\cdot\left(\frac{1}{\cos^2 x}\right)$

$f'(\frac{\pi}{4}) = -2\cdot\sin\frac{\pi}{4} - 3\cdot\left(\frac{1}{\cos^2\frac{\pi}{4}}\right)$

12 $f'(\frac{\pi}{4}) = -\cancel{2}\cdot\frac{\sqrt{2}}{\cancel{2}} - 3\cdot\frac{1}{\frac{2}{4}}$

$f'(\frac{\pi}{4}) = -\sqrt{2} - 6$

$f'(\frac{\pi}{4}) \doteq -7{,}41 \rightarrow$ funkcija je v tej točki padajoča,

saj je odvod v tej točki negativen

106/110

odl (5)

Super!! Pridno!