4 OR 200308

20 1) Izračunajte kot (na stotinko stopinje)pod katerim se sekata krivulji $2x^2 - y^2 = 5$ in $x^2 + y^2 = 4$ v drugem kvadrantu . Graf ! (6)

20 2) Izračunajte odvode :

a) $y = e^{2x} \cdot \cos(3x)$

b) $y = \ln(x^2 + 1)$

15 3) Dana je funkcija $f(x) = (x + a)^2$.Število a določite tako, da bo $f(2) = 1$ in $f'(2) < 0$!

20 4) Narišite graf funkcije $y = 1 - 2\sin x$ na intervalu $[-\pi, \pi]$. 8
Pod kakšnim kotom seka graf funkcije abscisno os ? 12

15 5) Ali obstajajo točke v katerih tangenta na krivuljo $y = \frac{x^2 - 6x + 13}{x - 3}$ oklepa z x osjo kot 45 ? (brez grafa) $(y' = 5^-)$

---

ponovitev – domača naloga

6 (2.) Izračunaj vrednost izraza: $|4 - 3i| + 29\, i\, (5 - 2i)^{-1} + i^{199} =$

3. Dani sta funkciji $f(x) = x + 1$ in $g(x) = x^2 - 4x + 5$.

10 (a) Natančno nariši grafa funkcij f in g v istem koordinatnem sistemu in izračunaj njuni presečišči.

(b) Zapiši enačbi tangent na funkcijo g v točkah z abscisama 4 in 1.

106

A+

20.3.2008

TEST A

Barbara Hudopisk
4.e

1. $K: x^2 + y^2 = 4$

$H: 2x^2 - y^2 = 5 \;/:5$

$\frac{2x^2}{5} - \frac{y^2}{5} = 1$

presečišča krivulj:

$x^2 + y^2 = 4$

$2x^2 - y^2 = 5$) +

$3x^2 = 9 \;/:3$

$x^2 = 3$

$x = \pm\sqrt{3} \quad y = \pm 1$

$P_1(\sqrt{3}, 1)$

$P_2(\sqrt{3}, -1)$

$P_3(-\sqrt{3}, 1)$

$P_4(-\sqrt{3}, -1)$ ✓

odvod krožnica:

$2x + 2y \cdot y' = 0$

$2y \cdot y' = -2x \;/: 2y$

$y' = \frac{-2x}{2y}$

$y_1' = -\frac{x}{y}$

odvod hiperbole:

$4x - 2y \cdot y' = 0$

$-2y \cdot y' = -4x \;/:(-2y)$

$y' = \frac{-4x}{-2y}$

$y_2' = \frac{2x}{y}$

$P_3(-\sqrt{3}, 1)$ - drugi kvadrant

$y_1' = \frac{-\sqrt{3}}{1}$

$y_1' = +\sqrt{3} = k_1$ ✓

$y_2' = \frac{2 \cdot (-\sqrt{3})}{1}$

$y_2' = -2\sqrt{3} = k_2$ ✓

$tg\varphi = \left| \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \right|$

$tg\varphi = \left| \frac{-2\sqrt{3} - \sqrt{3}}{1 + (+\sqrt{3}) \cdot (-2\sqrt{3})} \right|$ z besedo

$tg\varphi = \left| \frac{-\sqrt{3}}{1 + (-6)} \right|$

$tg\varphi = \frac{3\sqrt{3}}{5}$

$\varphi = 46°6'$

~~$\varphi = 19°6' = 19{,}11°$~~ //

graf:

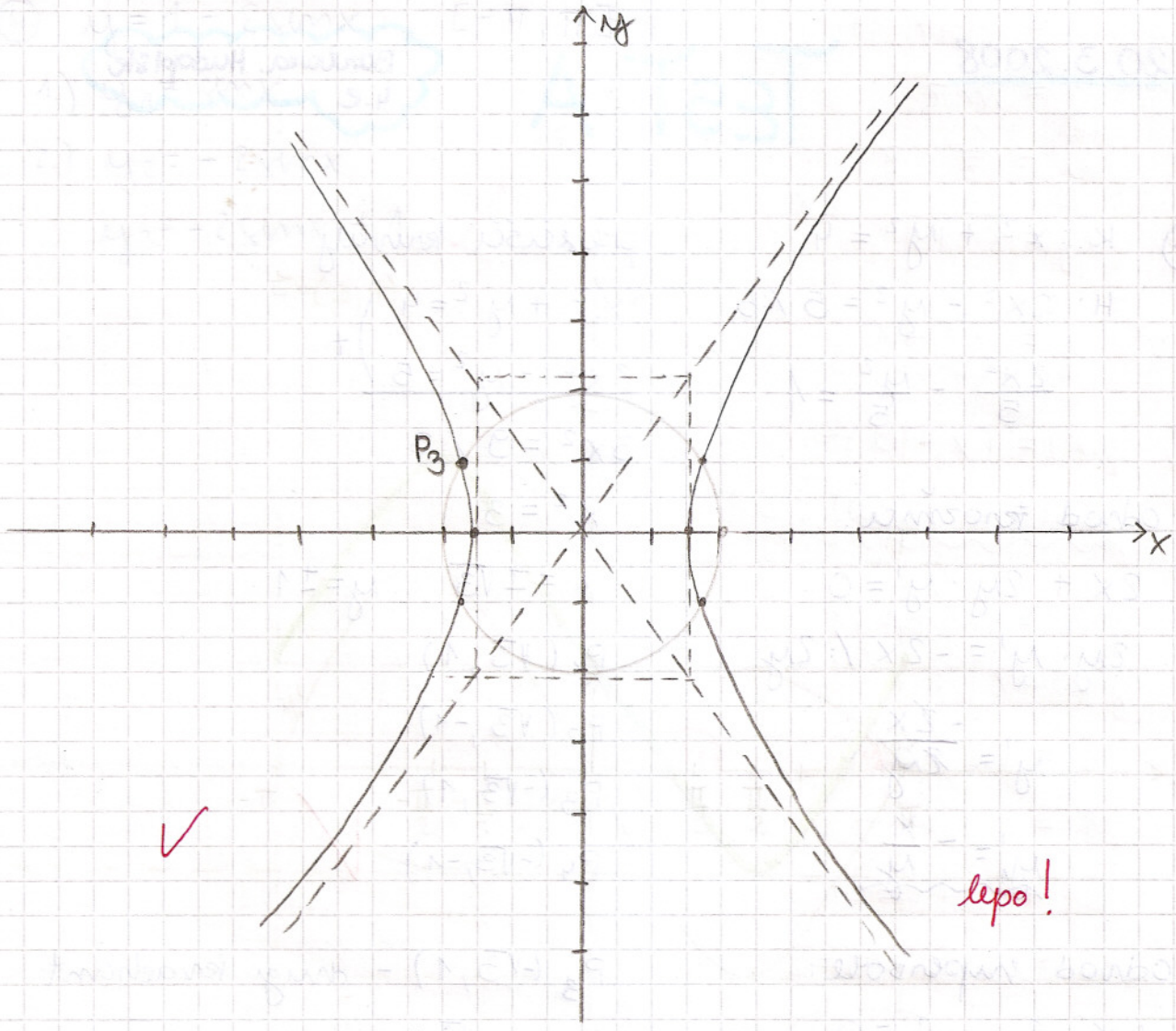
$K: x^2 + y^2 = 4$

$S(0,0) \quad r = 2$

$H: \frac{2x^2}{5} - \frac{y^2}{5} = 1$

$a^2 = \frac{5}{2} = \sqrt{\frac{5}{2}}$

$b^2 = 5 \Rightarrow b = \sqrt{5}$



18/20

✓

lepo!

② a) $y = e^{2x} \cdot \cos(3x)$

$y_1 = e^{2x}$ $\quad z = 2x \Rightarrow z' = 2$

$y_1 = e^{z}$ ✓

$y_1' = e^{z} \cdot z'$

$y_1' = e^{2x} \cdot 2$

$y_1' = 2e^{2x}$ ✓

$y_2 = \cos(3x)$ $\quad z = 3x$

$y_2 = \cos z$ $\quad z' = 3$

$y_2' = -\sin z \cdot z'$

$y_2' = -\sin 3x \cdot 3$

$y_2' = -3\sin 3x$ ✓

$y' = (e^{2x})' \cdot \cos(3x) + e^{2x} \cdot \cos(3x)'$

$y' = 2e^{2x} \cdot \cos(3x) + e^{2x} \cdot (-3\sin 3x)$

10

$y' = e^{2x} \cdot (2\cos 3x - 3\sin 3x)$ ✓

b) $y = \ln \cdot (x^2+1)$ $\quad z = x^2+1$

$y = \ln z$ $\quad z' = 2x$ ✓

$y' = \frac{1}{z} \cdot z'$

$y' = \frac{1}{x^2+1} \cdot 2x$

$y' = \frac{2x}{x^2+1}$ ✓

10

③ $f(x) = (x+a)^2$ $\quad a_1 = -3$, $a_2 = -1$

$f(2) = 1 \; ; \; f'(2) < 0$

$f(2) = (2+a)^2$

$f(2) = 1$

$1 = (2+a)^2$ ✓

$1 = 4 + 4a + a^2$

$a^2 + 4a + 3 = 0$

$(a+3)\cdot(a+1) = 0$ ✓

$a_1 = -3$ $\quad a_2 = -1$ → to je pravilna rešitev če upoštevamo ✓ pogoja, da je $f(2) = 1$ in $f'(2) < 0$.

$f(x) = x^2 + 2ax + a^2$

$f'(x) = 2x + 2a$

~~$f'(2) = 2 \cdot 2 + 2a$~~ ✓

~~$f'(2) = 6$~~

$2x + 2 < 0$

$2x < -2 \;/:2$

$x < -1$

$4 + 2a < 0$

$a < -2 \implies a = -3$ ✓

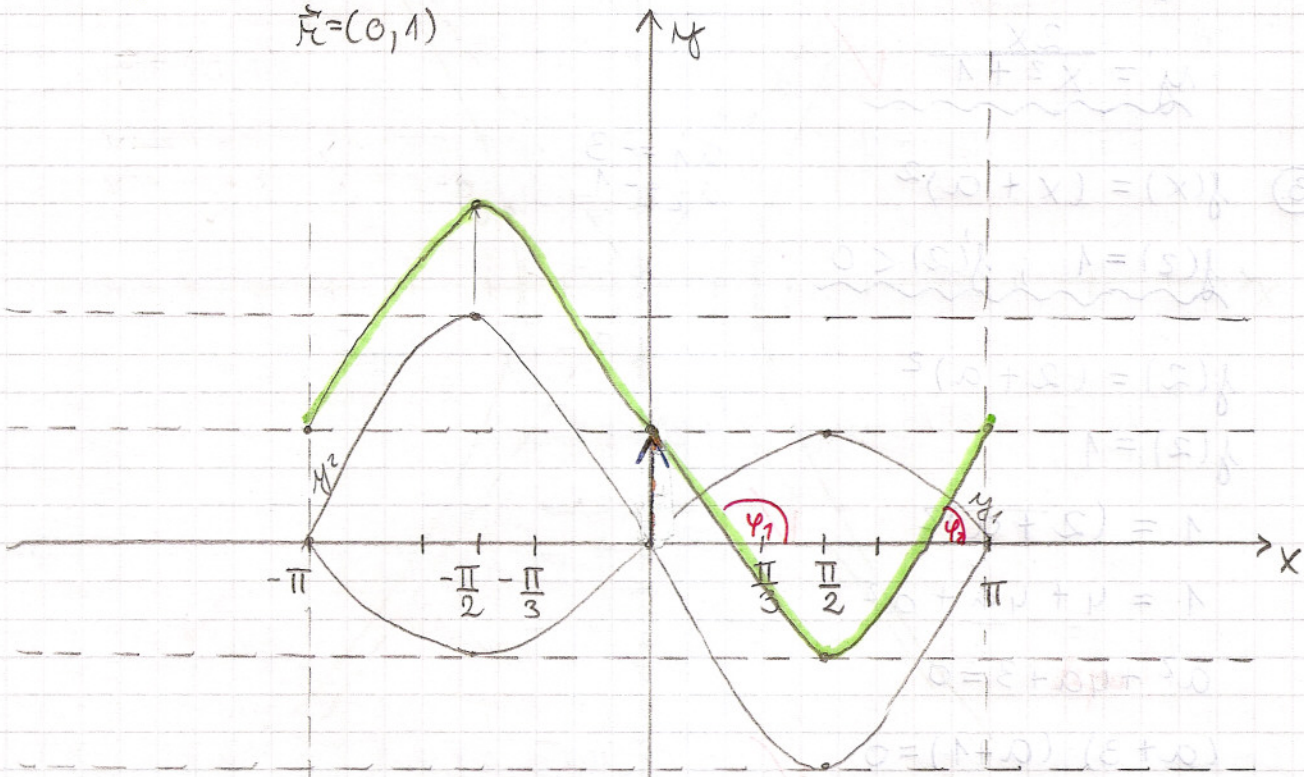
9/15

④ $y = 1 - 2\sin x \quad [-\pi, \pi]$

1.) $y_1 = \sin x$

2.) $y_2 = -2\sin x$

$y_3 = -2\sin x + 1$ ✓



presečišča z x-osjo: ($y=0$) ✓

$1 - 2\sin x = 0$

$-2\sin x = -1 \quad /:(-2)$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ ✓

$T_1\left(\frac{\pi}{6}, \frac{3-\pi}{3}\right)$

$y = 1 - 2\sin x$

$y' = -2\cos x$

$y' = -2\cos\left(\frac{\pi}{6}\right)$

$y' = -2 \cdot \frac{\sqrt{3}}{2}$

$y' = -\sqrt{3}$ ✓

$y' = k = \tan\varphi$

$y' = -\sqrt{3}$

$\tan\varphi = -\sqrt{3}$

$\varphi = 60°$

2) $x = \frac{5\pi}{6}$

$\varphi = 60°$

1) suhlasni kot

$\varphi = 120°$

glej graf!

14/20

⑤ $y = \frac{x^2-6x+13}{x-3}$ $\quad \varphi = 45°$

$\tan\varphi = k_t$ ✓

$\tan 45° = 1$

$\underline{k_t = 1}$ ✓

$y' = \frac{(2x-6)\cdot(x-3) - (x^2-6x+13)\cdot 1}{(x-3)^2}$

$y' = \frac{2x^2 - 6x - 6x + 18 - x^2 + 6x - 13}{(x-3)^2}$

$y' = \frac{x^2-6x+5}{(x-3)^2}$ ✓

$y'(1) = k_t$

$y' = 1$

$1 = \frac{x^2-6x+5}{(x-3)^2} \quad / \cdot (x-3)^2$ ✓

$x^2 - 6x + 9 = x^2 - 6x + 5$

$x^2 - 6x + 9 - x^2 + 6x - 5 = 0$

$9 - 5 = 0$

$4 = 0$ (je ni)

rešitev ne obstaja, zato točke v katerih tangenta na krivuljo y, oklepa ~~kot~~ z x-osjo kot 45° ne obstajajo! ✓

15/15-

⑥ $|4-3i| + 29i\cdot(5-2i)^{-1} + i^{199} =$

$= \sqrt{(4-3i)\cdot(4+3i)} + 29i\cdot\frac{1}{5-2i} + i^3 =$

$= \sqrt{16 - 3i^2} + \frac{29i}{5-2i} + (-i) =$

$= \sqrt{16+9} + \frac{29i\cdot(5+2i)}{(5-2i)\cdot(5+2i)} - i =$ ✓ ✓

$= 5 + \frac{145i + 58i^2}{25 - 4i^2} - i =$

$= 5 + \frac{145i + 58\cdot(-1)}{29} - i = 5 + \frac{145i - 58}{29} - i = \frac{145 + 145i - 58 + 29i}{29} =$ 6/6

$= \frac{87 + 116i}{29} = \frac{29\cdot(3+4i)}{29} = 3 + 4i$ ✓

(7) $f(x) = x + 1$

$g(x) = x^2 - 4x + 5$

presečišči:

$f(x) = g(x)$ ✓

$x + 1 = x^2 - 4x + 5$

$x^2 - 5x + 4 = 0$

$(x-1) \cdot (x-4) = 0$

$x_1 = 1 \quad x_2 = 4$

$y_1 = 2 \quad y_2 = 5$

✓

$P_1(1,2)$

$P_2(4,5)$ ✓

ničle: $x^2 - 4x + 5 = 0$

ni realnih ničel

$D = b^2 - 4ac$

$D = 16 - 4 \cdot 1 \cdot 5$

$D = 16 - 20$

$D = -4$

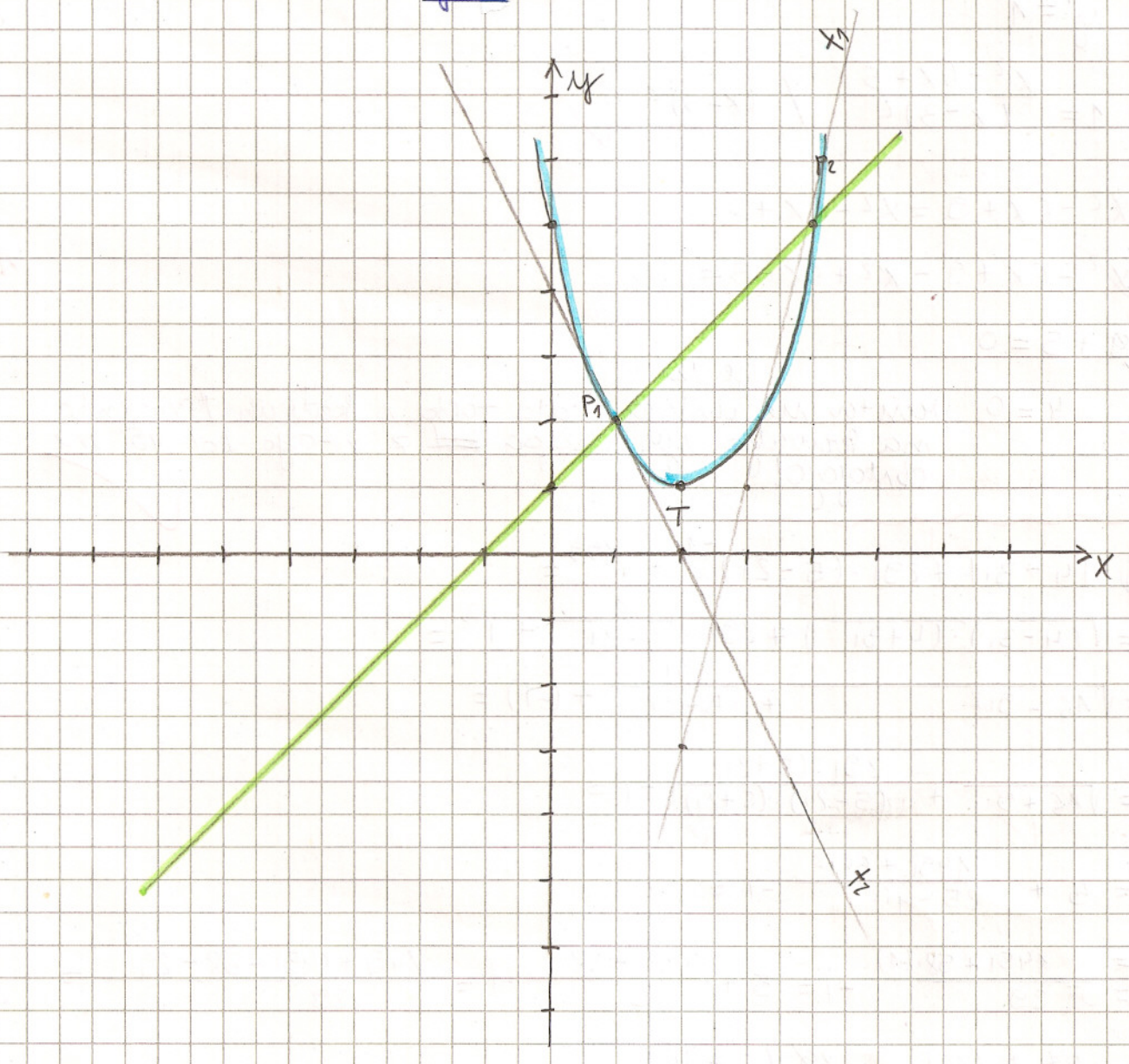
tečne: $p = \frac{-b}{2a} = \frac{4}{2} = \underline{2}$

$q = \frac{-D}{4a} = \frac{4}{4} = \underline{1}$

$T(2,1)$

zač. vrednost: $(x = 0)$

$\underline{y = 5}$



$g(x) = x^2 - 4x + 5$

$x_1 = 4 \Rightarrow y_1 = 5$

$x_2 = 1 \Rightarrow y_2 = 2$

$T_1(4,5)$

$T_2(1,2)$

$g'(x) = 2x - 4$

1.) $g'(4) = 2 \cdot 4 - 4$

$g'(4) = 4$

$g'(x) = k_{t} = 4$

$k_{t_1} = 4$

$T_1(4,5)$

$y - y_1 = k_{t_1} \cdot (x - x_1)$

$y - 5 = 4 \cdot (x - 4)$

$y = 4x - 16 + 5$

$y_1 = 4x - 11$ ✓

2.) $T_2(1,2)$

$g'(1) = 2 \cdot 1 - 4$

$g'(1) = -2$

$g'(x) = -2 = k_{t_2}$

$k_{t_2} = -2$

$y - y_2 = k_{t_2} \cdot (x - x_2)$

$y - 2 = -2 \cdot (x - 1)$

$y = -2x + 2 + 2$

$y_2 = -2x + 4$ ✓

10/10

92/106

odl (5)