VZ:

metoda nedoločnih koeficientov

$(x-3) = A(x+1) + B(x-1)$

$x-3 = Ax + A + Bx - B$

$1\,x - 3 = x(A+B) + (A-B)$

primerjava

$A+B=1$
$A-B=-3$)+

$2A = -2$
$A = -1 \quad B = 2$

$\frac{x-3}{(x-1)(x+1)} = \frac{-1}{x-1} + \frac{2}{x+1}$

① $\int \frac{x^3-3}{x^2-1}\,dx \overset{*}{=}$

$(x^3-3) : (x^2-1) = x \sim$ kol.

ost $= (x-3)$

$\overset{*}{=} \int x\,dx + \int \frac{x-3}{x^2-1}\,dx$

↳ nazaj na

$/\cdot(x-1)(x+1)$

$\frac{x-3}{x^2-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)} \quad /\cdot()$

nadaljevanje

$= \int \left(x + \frac{-1}{x-1} + \frac{2}{x+1}\right) dx = \int x\,dx - \int \frac{1}{x-1}\,dx + 2\int \frac{1}{x+1}\,dx =$

$= \frac{1}{2}x^2 - \ln|x-1| + 2\ln|x+1| + C = \frac{1}{2}x^2 + \ln\frac{(x+1)^2}{x-1} + C$

VAJA:

2. ~~Z~~: $\int \frac{x^2-9x+17}{x^3-3x^2+4}\,dx =$ ...

razbijemo

$\frac{x^2-9x+17}{x^3-4x^2+x^2+4} = \frac{x^2-9x+17}{-4(x^2-1)+x^2(x+1)} = \frac{x^2-9x+17}{-4(x-1)(x+1)+x^2(x+1)} = \frac{x^2-9x+17}{(x+1)(-4x+4+x^2)} =$

$= \frac{x^2-9x+17}{(x-2)^2(x+1)} = \frac{A}{(x-2)^2} + \frac{B}{(x-2)} + \frac{C}{(x+1)} \quad /\cdot(x-2)^2(x+1)$

VSE MOŽNOSTI!

VIŠJA STOPNJA → VSE STOPNJE DO TISTE

$x^2-9x+17 = A(x+1) + B(x^2-x-2) + C(x^2-4x+4)$

$x^2-9x+17 = Ax + A + Bx^2 - Bx - 2B + Cx^2 - 4Cx + 4C$

$B+C=1 \quad \Rightarrow B = 1-C$
$A-B-4C=-9$
$A-2B+4C=17$

$A - 1 + C - 4C = -9$
$A - 2 + 2C + 4C = 17$

$A - 3C = -8$
$A + 6C = 19$)−

$-9C = -27$

$C = 3, \quad B = 1-3 = -2 \quad A = 3\cdot 3 - 8 = 1$

$t = x-2$ $\int \frac{1}{t^2}\,dt$

$\int \left(\frac{1}{(x-2)^2} - \frac{2}{(x-2)} + \frac{3}{(x-1)}\right) dx = -1\,(x-2)^{-1} - 2\ln|x-2| + 3\ln|x-1| + C =$

$= -\frac{1}{x-2} + \ln\left|\frac{(x-1)^3}{(x-2)^2}\right| + C$

$\frac{\sim}{x^4(x-3)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{(x-3)} + \frac{F}{(x-3)^2}$ razbijemo do konca!

ali

$\frac{\sim}{x^4(x-3)^2} = \frac{Ax^3+Bx^2+Cx+D}{x^4} + \frac{Ex+F}{(x-3)^2}$

$\frac{\sim\sim}{(x-2)x^2(x+1)^3} = \frac{A}{x-2} + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{x+1} + \frac{E}{(x+1)^2} + \frac{F}{(x+1)^3}$

$\frac{\sim}{(x-3)(x^2+1)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+1}$ ← števec nižje stopnje

↳ker je kvadratni!

Mastarbi

3) $\int \frac{8}{x^3-4x}\,dx = *$

$\frac{8}{x^3-4x} = \frac{8}{x(x^2-4)} = \frac{8}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$

$8 = A(x^2-4) + B(x^2+2x) + C(x^2-2x)$

$A+B+C=0$
$2B-2C=0$
$-4A=8 \Rightarrow A=-2$

$B+C=2 \mid \cdot 2$
$2B-2C=0$

$2B+2C=4$
$2B-2C=0$)+
$4B=4$
$B=1$

$2B=2C$
$B=C \Rightarrow C=1$

$= * \int \left(-\frac{2}{x} + \frac{1}{x-2} + \frac{1}{x+2}\right) dx = -2\int x\,dx + \int (x-2)^{-1}dx +$

$\int (x+2)^{-1}dx = -2\ln x + \ln|x-2| + \ln|x+2| + C =$

$= -\ln x^2 + \ln|x^2-4| + C = \ln\left|\frac{x^2-4}{x^2}\right| + C$

4) $\int \frac{10}{x^3+x^2+9x+9}\,dx =$

$$\frac{10}{x^3+x^2+9x+9} = \frac{10}{x^2(x+1)+9(x+1)} = \frac{10}{(x+1)(x^2+9)} =$$

$$= \frac{A}{x+1} + \frac{Bx+C}{x^2+9}$$

$$10 = A(x^2+9) + (Bx+C)(x+1)$$

$$10 = Ax^2 + 9A + Bx^2 + Cx + Bx + C$$

$$A+B=0$$
$$C+B=0$$
$$9A+C=10 \Rightarrow C = 10-9A$$

$$10-9A+B=0$$
$$-9A+B=-10$$

$$\frac{1}{a}\,\text{arc tg}\,\frac{x}{a} + C$$

$$-9A+B=-10$$
$$A+B=0 \quad (-)$$

$$-10A=-10$$
$$A=1, \quad B=-A=-1, \quad C=10-9=1$$

$$\int\left(\frac{1}{x+1}+\frac{-x+1}{x^2+9}\right)dx = \int\left(\frac{1}{x+1}\right)dx - \int\frac{x-1}{x^2+9}\,dx =$$

$$= \ln|x+1| + C - \int\frac{x-1}{x^2+9}\,dx = \ln|x+1| - \int\frac{x\,dx}{x^2+9} + \int\frac{1}{x^2+9}\,dx =$$

$x^2+9=z$

$$= \ln|x+1| - \frac{1}{2}\int\frac{2x}{x^2+9}\,dx$$

$$= \ln|x+1| - \frac{1}{2}\ln|x^2+9| + \frac{1}{3}\,\text{arc tg}\,\frac{x}{3} + C$$

$$\int\frac{dx}{x^2+9} = \int\frac{1}{9\left(1+\frac{x^2}{9}\right)}\,dx = \frac{1}{9}\int\frac{1}{1+\left(\frac{x}{3}\right)^2}\,dx =$$

$$\int\frac{dx}{1+x^2} = \text{arctg}\,x + C \qquad z=\frac{x}{3} \qquad dz=\frac{dx}{3}$$

$$= \frac{1}{9}\int\frac{3\,dz}{1+z^2} = \frac{1}{3}\,\text{arctg}\,z + C$$

$$= \frac{1}{3}\,\text{arctg}\,\frac{x}{3} + C$$

$$\int\frac{dx}{x^2+a^2} = \frac{1}{a}\,\text{arctg}\,\frac{x}{a} + C \qquad \int\frac{dx}{x^2+3} = \frac{1}{\sqrt{3}}\,\text{arctg}\,\frac{x}{\sqrt{3}} + C$$

KVADRATNI NERAZCEPNI → če je stvar ... → arctg...