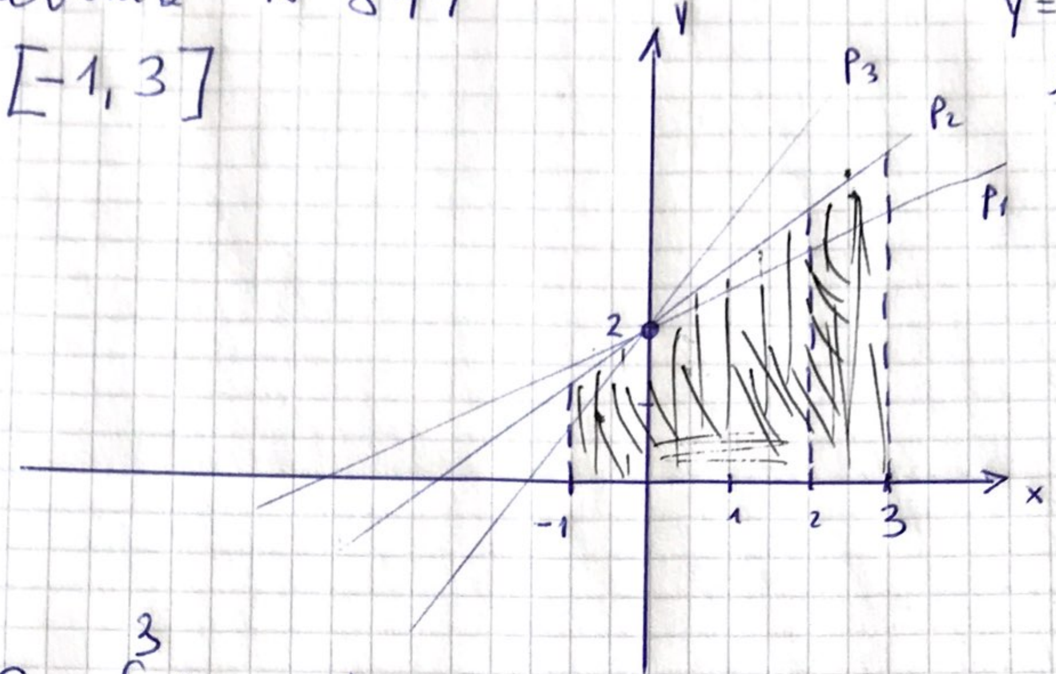
Učbenik N 841

$[-1, 3]$

$y = kx + 2$

šop premic



$$S = \int_{-1}^{3} (kx + 2)\,dx$$

$$14 = \left(k\frac{x^2}{2} + 2x \right)\Big|_{-1}^{3} \longrightarrow 14 = \frac{9}{2}k + 6 - \left(\frac{k}{2} - 2\right)$$

$$\cancel{14 = \frac{9k}{2} + 6 + \left(\frac{k}{2} - 2\right) / \cdot 2}$$

$$\boxed{k = \frac{3}{4}}$$

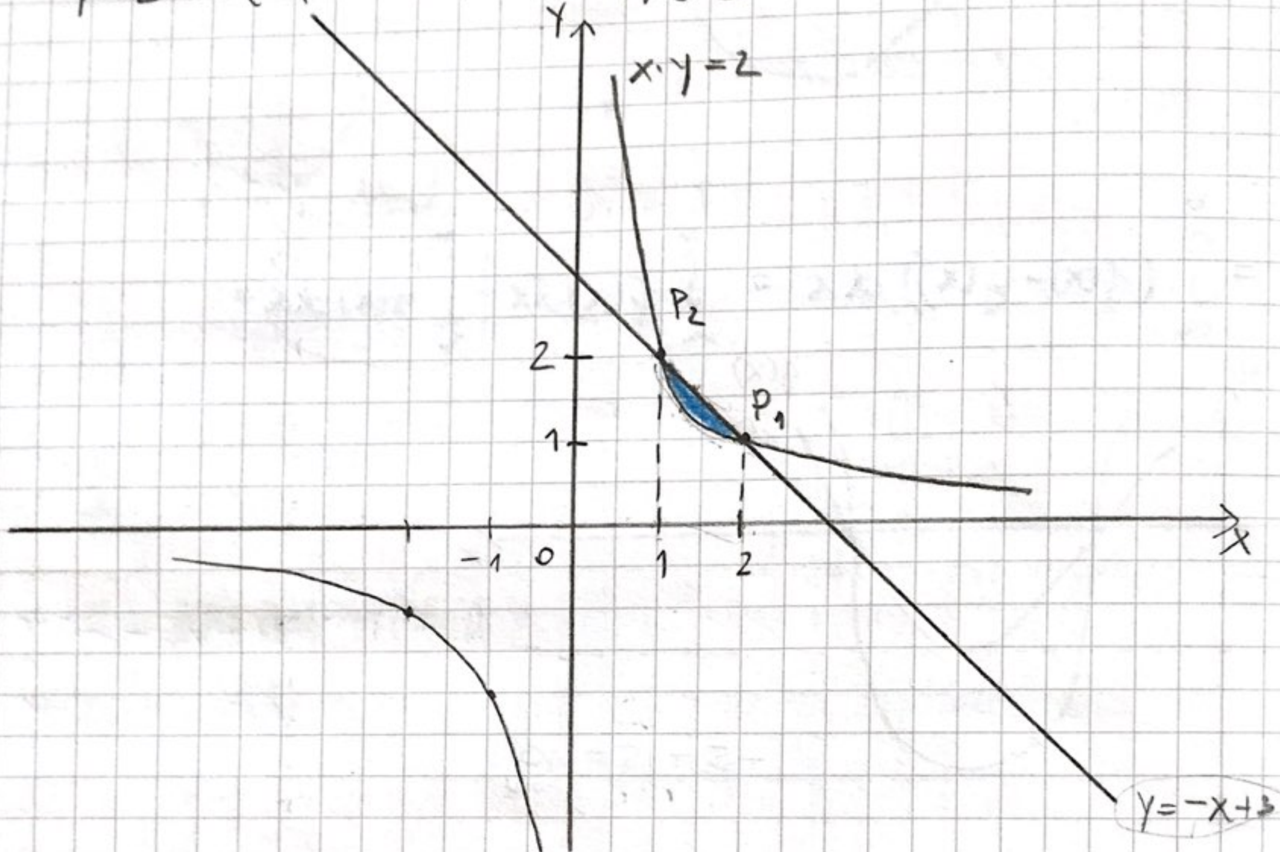
4. Izračunaj ploščino lika, ki ga omejujeta krivulja $x \cdot y = 2$ in premica $x + y = 3$. Nariši ustrezno sliko.

$x \cdot y = 2$

$y = 2x^{-1}$ (hiperbola)

$x + y = 3$

$y = -x + 3$



Presečišče:

$[1,2]$

$x \cdot y = 2$

$x + y = 3 \Rightarrow x = 3 - y$

$(3-y)y = 2$

$3y - y^2 = 2$

$y^2 - 3y + 2 = 0$

$(y-1)(y-2) = 0$

$y_1 = 1, \ y_2 = 2$

$x_1 = 3 - 1 = 2$

$x_2 = 3 - 2 = 1$

$P_1(2,1)$

$P_2(1,2)$

→

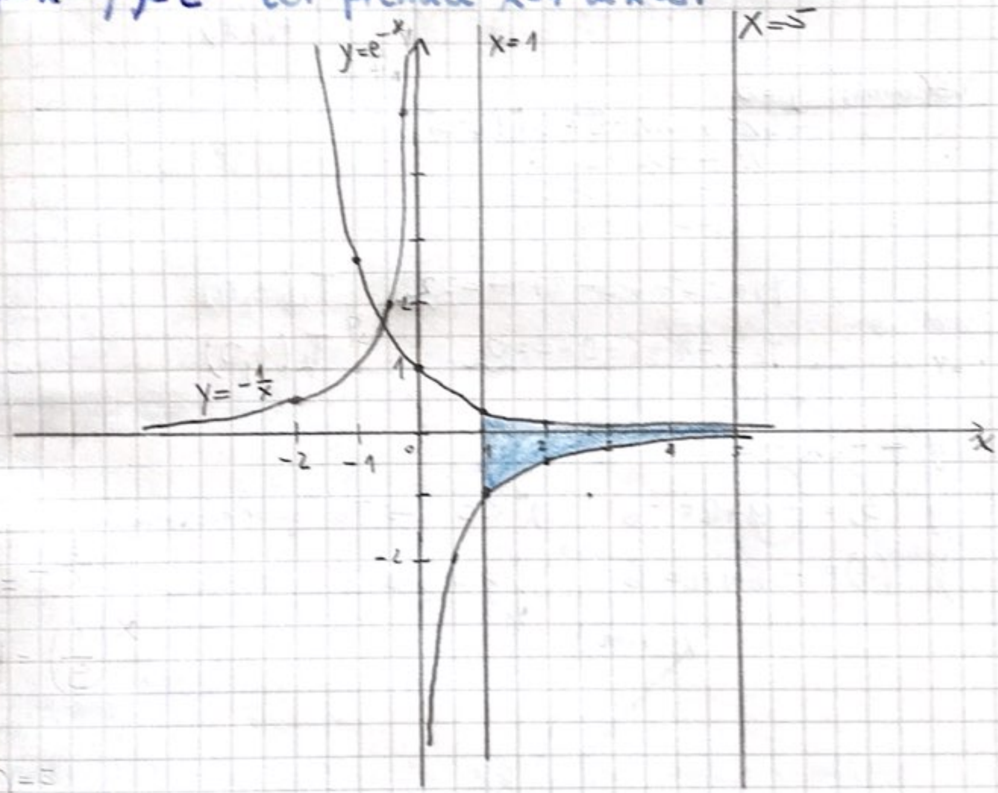
$$S=\int_1^2\left(-x+3-\frac{2}{x}\right)dx=$$

$$=\int_1^2\left(-x-2x^{-1}+3\right)dx=$$

$$=\left(-\frac{x^2}{2}-2\ln|x|+3x\right)\Big|_1^2=\left(-\frac{2^2}{2}-2\ln|2|+3\cdot 2\right)-\left(-\frac{1}{2}+3\right)=$$

$$=\left(-2-\ln 4+6+\frac{1}{2}-3\right)=\frac{3}{2}-\ln 4\doteq 0{,}1137$$

12. Izračunaj ploščino lika, ki ga omejujejo krivulji $y=-x^{-1}$, $y=e^{-x}$ ter premici $x=1$ in $x=5$.



$$S=\int_a^b\left(f(x)-g(x)\right)dx$$

$$S=\int_1^5\left(e^{-x}+\frac{1}{x}\right)dx=\left(-e^{-x}+\ln|x|\right)\Big|_1^5=$$

$$=\left(-e^{-5}+\ln 5+e^{-1}-\ln 1\right)=$$

$$=-\frac{1}{e^5}+\frac{1}{e}+\ln 5=\frac{-1+e^4}{e^5}+\ln 5\doteq 1{,}9706$$

3. Izračunaj ploščino lika, ki ga omejujeta krivulja $y=3x-x^2$ in premica $y=x-3$. Nariši ustrezno sliko.

$y=-x^2+3x$

$y'=-2x+3$

$y'=-2x+3=0$

$2x=3$

$x=\frac{3}{2}$ $\Rightarrow$ Teme $T\left(\frac{3}{2}, \frac{9}{4}\right)$

$y=-\left(\frac{3}{2}\right)^2+3\cdot\frac{3}{2}$

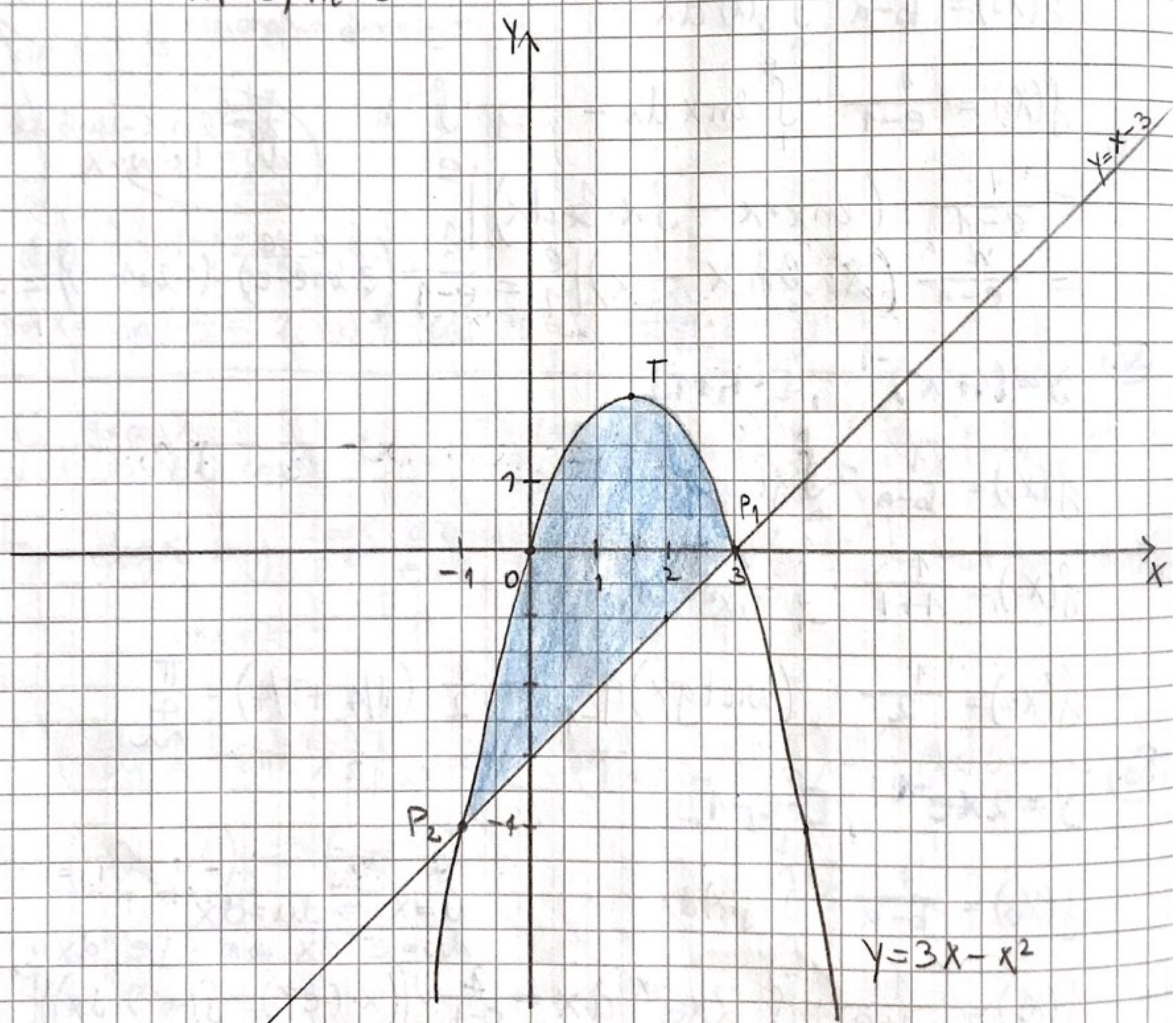
$y=-\frac{9}{4}+\frac{9}{2}=\frac{9}{4}$

ničle: $-x^2+3x=0 \quad /\cdot(-1)$

$x^2-3x=0$

$x(x-3)=0$

$x_1=0, \; x_2=3$



PRESEČIŠČE:

$y = x - 3$, $y = -x^2 + 3x$

$y = y$

$-x^2 + 3x = x - 3$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x_1 = 3$, $x_2 = -1$

$y_1 = 0$

$y_2 = -4$

$P_1(3, 0)$

$P_2(-1, -4)$

$$S = \int_a^b (f(x) - g(x))\,dx$$

$$S = \int_{-1}^{3} (3x - x^2 - x + 3)\,dx = \int_{-1}^{3} (-x^2 + 2x + 3)\,dx = \left(-\frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}} + \frac{3x}{1}\right)\Big|_{-1}^{3} =$$

$$= \left(-\frac{3^3}{3} + 3^2 + 3 \cdot 3\right) - \left(-\frac{(-1)^3}{3} + (-1)^2 - 3\right) = (-\cancel{9} + \cancel{9} + 9) - \left(\frac{1}{3} + 1 - 3\right) =$$

$$= 9 - \left(\frac{1}{3} - 2\right) = \frac{27}{3} - \frac{1}{3} + \frac{6}{3} = \frac{32}{3}\,e^2$$

6. Izračunaj ploščino lika, ki ga omejujeta krivulja $y = x^2 - 4x + 5$ in pozitivna veja hiperbole $xy = 2$. Nariši ustrezno sliko. Iz ploščine ugotovi medsebojno lego krivulj.

$y = x^2 - 4x + 5$

ničle: $x^2 - 4x + 5 = 0$

$D = (-4)^2 - 4 \cdot 1 \cdot 5 = 16 - 20 = -4$ ni realnih ničel

teme: $y' = 2x - 4 = 0$

$x - 2 = 0$

$x = 2$, $y = 2^2 - 4 \cdot 2 + 5 = 4 - 8 + 5 = 1$ $T(2, 1)$

hiperbola:

$x \cdot y = 2$

$y = 2x^{-1}$

PRESEČIŠČE:

$x \cdot y = 2$ $y = x^2 - 4x + 5$

$y = \frac{2}{x}$

$\frac{2}{x} = x^2 - 4x + 5 \quad / \cdot x$

$x^3 - 4x^2 + 5x - 2 = 0$

	1	-4	5	-2
		1	-3	2
1	1	-3	2	0

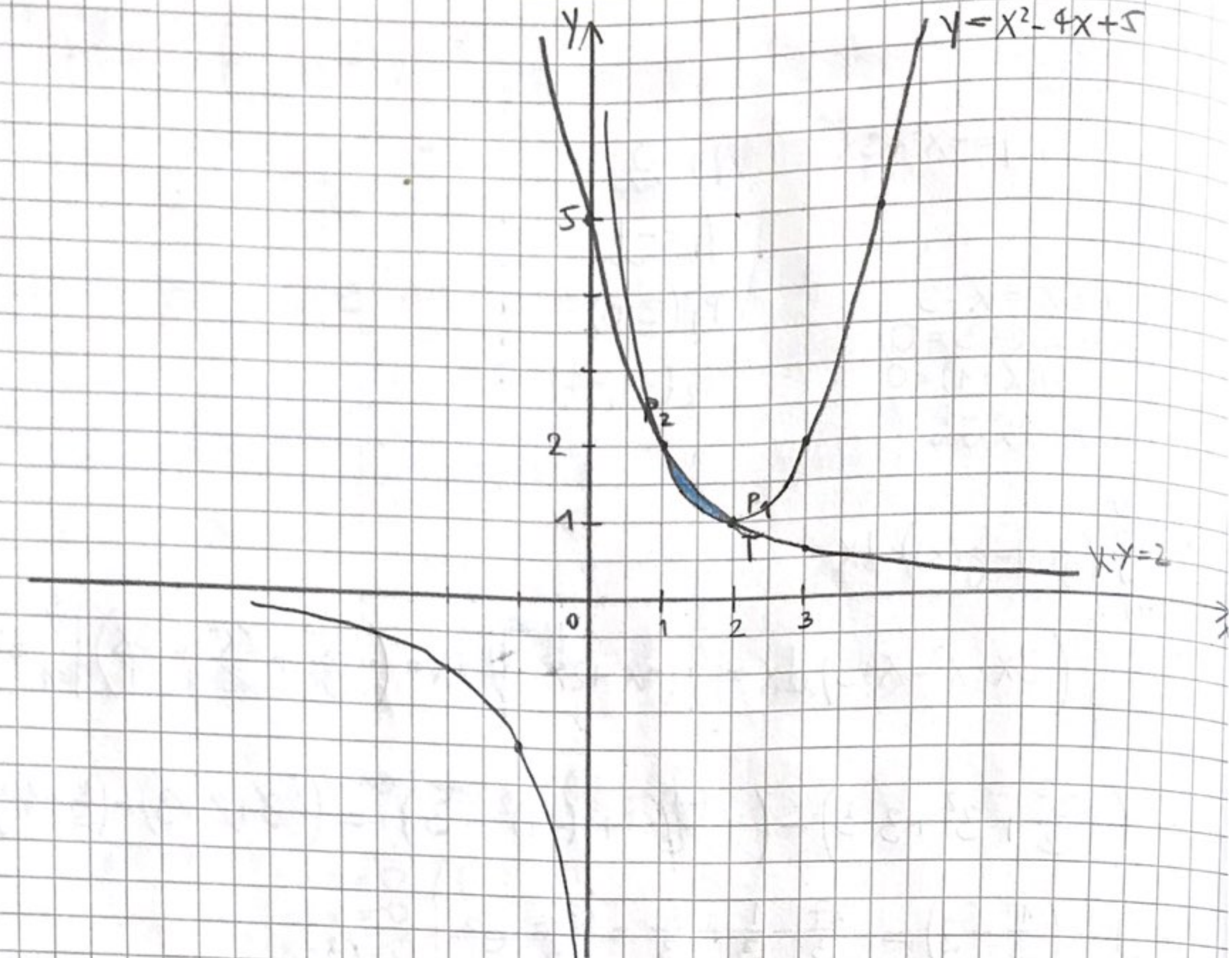
$x_1 = 1$, $y_1 = 2$ $P_1(1, 2)$

$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x_2 = 2$, $x_3 = 1$

$y_2 = 1$ $P_2(2, 1)$



$$S = \int_a^b \left(f(x) - g(x)\right) dx$$

$$S = \int_1^2 \left(\frac{2}{x} - x^2 + 4x - 5\right) dx = \left(2\ln|x| - \frac{x^3}{3} + \frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}} - 5x\right)\Big|_1^2 =$$

$$= \left(2\ln(2) - \frac{2^3}{3} + 2\cdot 2^2 - 5\cdot 2\right) - \left(2\ln(1) - \frac{1^3}{3} + 2\cdot 1^2 - 5\cdot 1\right) =$$

$$= \left(2\cdot\ln 2 - \frac{8}{3} + 8 - 10\right) - \left(-\frac{1}{3} + 2 - 5\right) =$$

$$= 2\cdot\ln 2 - \frac{8}{3} \underline{- 2} + \frac{1}{3} \underline{- 2} + 5 = \ln 4 - \frac{7}{3} + \frac{3}{3} = \ln 4 - \frac{4}{3} =$$

$$= \underline{0{,}053}$$

→ Ker ima ploščina pozitivni predznak, pomeni, da poteka hiperbola nad parabolo.