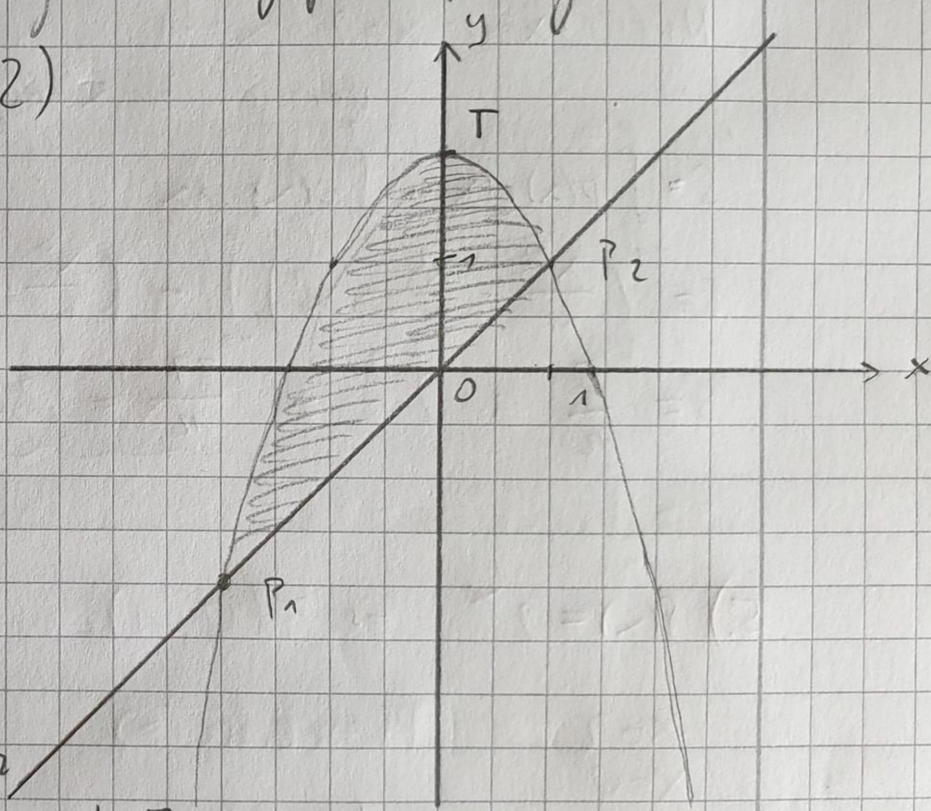
Domača Naloga => 1 list (OR)

1.) Izračunajte ploščino lika ki ga omejijeta funkciji =>

$y = 2 - x^2 = (\sqrt{2} - x)(\sqrt{2} + x) \Rightarrow T(0, 2)$

$y = x$



Presečišče => $y = y$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2 \quad y = -2 \quad P_1(-2, -2)$

$x = 1 \quad y = 1 \quad P_2(1, 1)$

$$S = \int_{-2}^{1} (2 - x^2 - x)\,dx = \left(2x - \frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{-2}^{1}$$

$$= \left(1\frac{1}{6}\right) - \left(-3\frac{1}{3}\right) = 4{,}5$$

2.) Enako narodilo, kot prej

$f(x) = \frac{5}{9}x^2 - 1 = \left(\frac{\sqrt{5}}{3}x - 1\right)\left(\frac{\sqrt{5}}{3}x + 1\right) \Rightarrow T_1(0, -1)$

$g(x) = \frac{1}{3}x^2 + 1 \Rightarrow T_2(0, 1)$

Presečišče $\Rightarrow f(x) = g(x)$

$\frac{5}{9}x^2 - 1 = \frac{1}{3}x^2 + 1 \quad / \cdot 9$

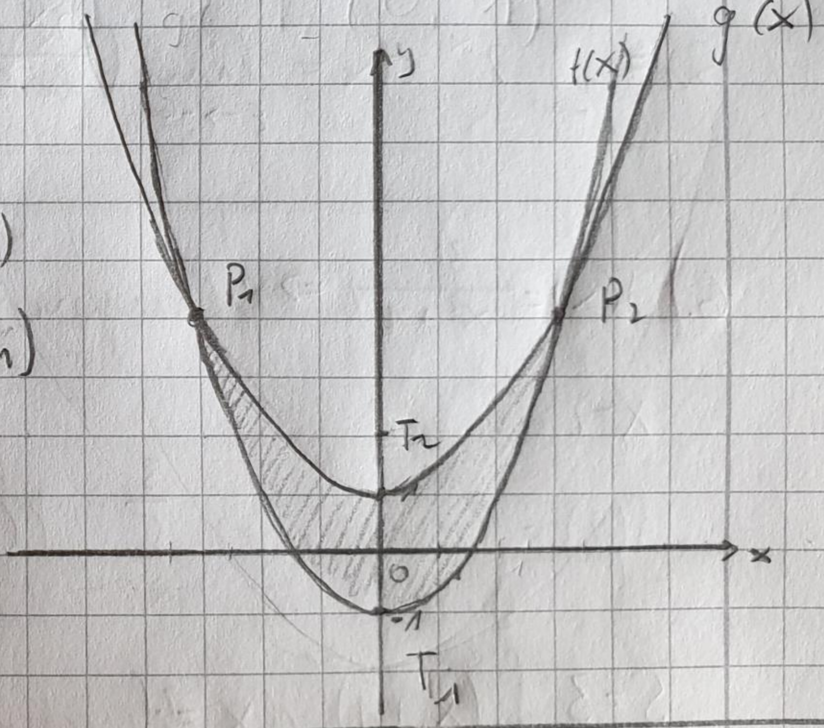
$2x^2 = 18 \Rightarrow x_{1,2} = \pm 3 \quad y_{1,2} = 4$

$P_2(3, 4)$, $P_1(-3, 4)$

$S = \int_{-3}^{3} (g(x) - f(x))\,dx = 2 \cdot \int_{0}^{3} (g(x) - f(x))\,dx$

$S = 2 \cdot \int_{0}^{3} \left(-\frac{2}{9}x^2 + 2\right)dx = 2 \cdot \left(-\frac{2x^3}{27} + 2x\right)\Big|_0^3$

$S = 2 \cdot 4 = \boxed{8}$



3.) $y_1 = \sqrt{x}$

$y_2 = \frac{1}{x}$

$y_3 = 2\sqrt{2x}$

Presečišča $\Rightarrow$

$y_1 = y_2$

$\sqrt{x} = x^{-1} \quad /^2$

$x - x^{-2} = 0$

$(x^3 - 1) : x^{+2} = 0$

$x = 1 \Rightarrow y = 1$

$y_1 = y_3$

$\sqrt{x} = 2\sqrt{2x} \quad /^2$

$x = 8x$

$x = 0 \Rightarrow y = 0$

$y_2 = y_3$

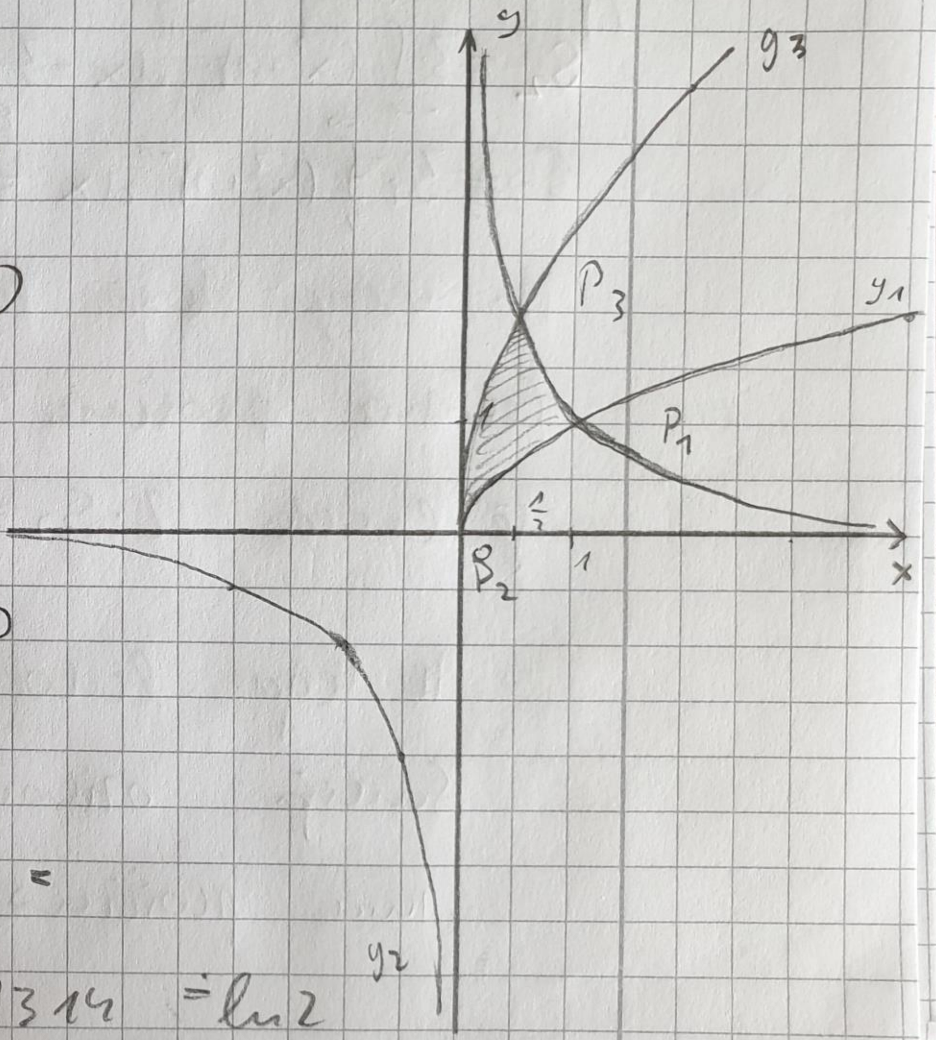
$x^{-1} = 2\sqrt{2x} \quad /^2$

$8x - x^{-2} = 0$

$(8x^3 - 1) : x^{+2} = 0$

$x = \frac{1}{2} \quad y = 2$

$P_1(1, +1); P_2(0, 0); P_3(\frac{1}{2}, 2)$



$$S = \int_0^{\frac{1}{2}} (2\sqrt{2x} - \sqrt{x})\,dx + \int_{\frac{1}{2}}^{1} \left(\frac{1}{x} - \sqrt{x}\right) dx$$

$$S = \left(\frac{4\sqrt{2}}{3} x^{\frac{3}{2}} - \frac{2}{3} x^{\frac{3}{2}}\right) \Big|_0^{0,5} + \left(\ln|x| - \frac{2}{3} x^{\frac{3}{2}}\right) \Big|_{0,5}^{1} =$$

$$S = \left(\frac{4 - \sqrt{2}}{6}\right) + \left(-\frac{2}{3}\right) - (-0,9288) = 0,69314 \doteq \ln 2$$

4.) $f(x) = x^3 - x^2 - 2x = x(x^2 - x - 2) = x(x-2)(x+1)$

$f'(x) = 3x^2 - 2x - 2 = 0 \quad \sqrt{D} = 2\sqrt{7} \Rightarrow x_1 = \frac{1+\sqrt{7}}{3} \quad y \doteq -2,11 \Rightarrow T_1$

$x_2 = \frac{1-\sqrt{7}}{3} \quad y = 0,63 \Rightarrow T_2$

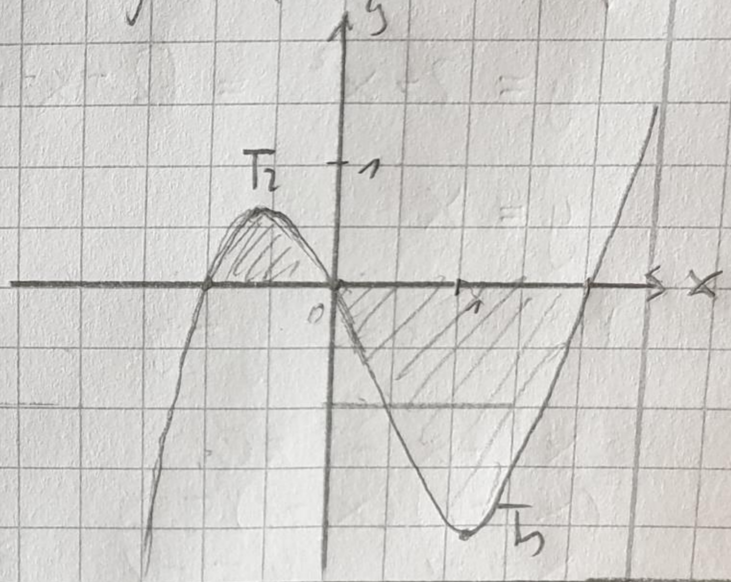
Ničle $\Rightarrow x=0 \quad x=2 \quad x=-1$

$\rightarrow$ ker je pod x-osjo

$S = \int_{-1}^{0} p(x)\,dx - \int_{0}^{2} p(x)\,dx$

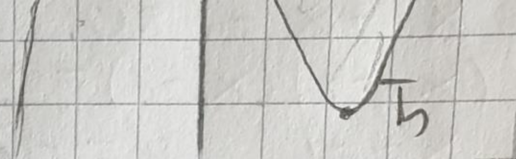
$S = \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2\right)\Big|_{-1}^{0} - \left(\frac{x^4}{4} - \frac{x^3}{3} - x^2\right)\Big|_{0}^{2}$

$S = \frac{5}{12} + 2\frac{2}{3} = \frac{37}{12} = 3\frac{1}{12}$



5.) $f(x) = 2^x \Rightarrow T(0,1)$

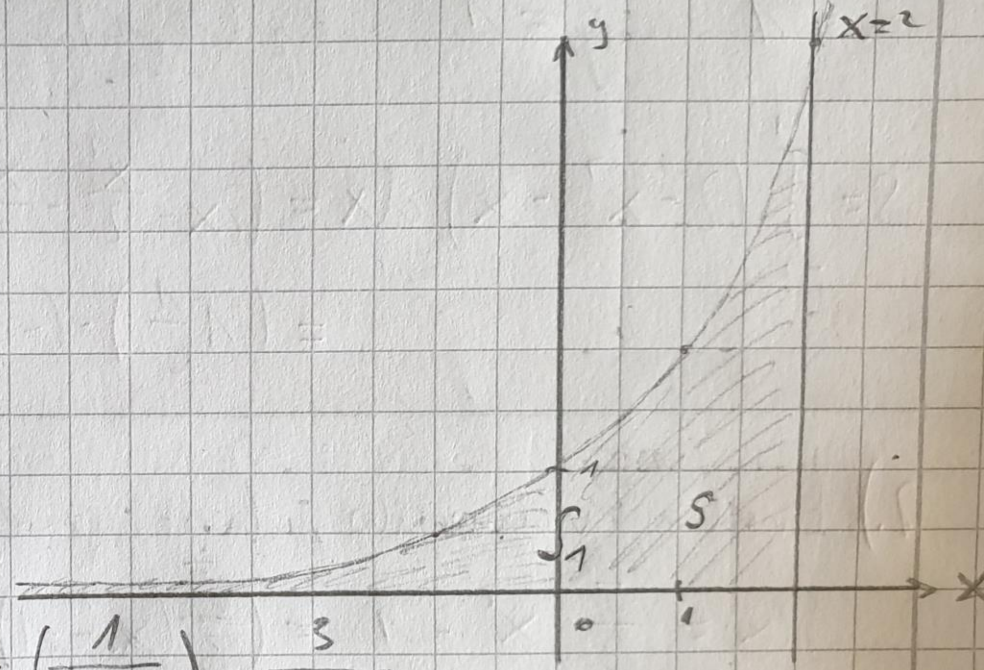
$S = \frac{\ldots}{12} + 2\frac{\ldots}{3} = \frac{\ldots}{12} = 3\frac{\ldots}{12}$



5.) $f(x) = 2^x \Rightarrow T(0,1)$

$x = 2$ Presječište $\Rightarrow$
$x = 0$

uvrstimo u $f(x)$ i dobimo $\begin{cases} P_1(0,1) \\ P_2(2,4) \end{cases}$

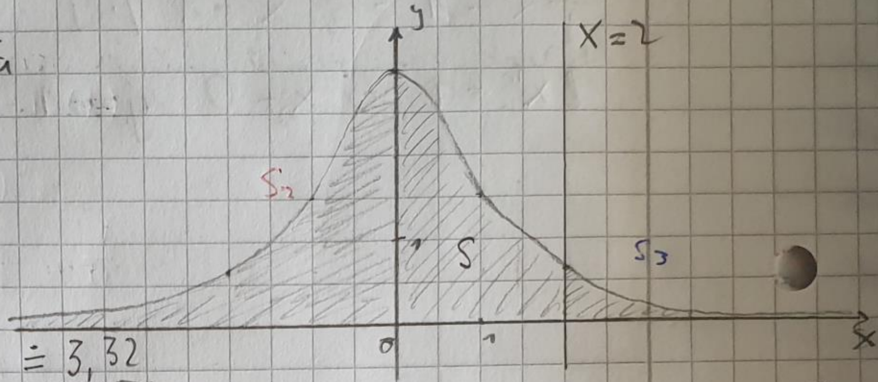


$$S = \int_0^2 2^x\,dx = \left(\frac{2^x}{\ln 2}\right)\Big|_0^2 = \left(\frac{4}{\ln 2}\right) - \left(\frac{1}{\ln 2}\right) = \frac{3}{\ln 2}$$

$$S_1 = \int_{-\infty}^{2} 2^x\,dx = \lim_{\varepsilon \to -\infty}\left(\frac{2^x}{\ln 2}\right)\Big|_{\varepsilon}^{2} = \left(\frac{4}{\ln 2}\right) - \underbrace{\left(\frac{2^{\varepsilon}}{\ln 2}\right)}_{\to 0} = \frac{4}{\ln 2}$$

x=2

6.) $f(x) = \frac{3}{x^2+1}$ => 3·Gausova krivulja



$x = 2$

$f(2)$ = Presečišče => $\frac{3}{5}$ $P(2, \frac{3}{5})$

$S = 3\int_0^2 (x^2+1)^{-1}\,dx = 3\cdot(\arctan x)\Big|_0^2 \doteq 3{,}32$

$S_2 = 3\cdot\int_{-\infty}^{0} (x^2+1)^{-1}\,dx = 3\lim_{\varepsilon\to-\infty} (\arctan x)\Big|_{\varepsilon}^{0} = 3\cdot\lim_{\varepsilon\to-\infty}(\arctan 0 - \arctan \varepsilon) = \frac{3\pi}{2}$

$S_3 = 3\cdot\int_2^{\infty} (x^2+1)^{-1}\,dx = S_2 - S = \frac{3\pi}{2} - 3{,}32 \doteq 1{,}39$

Iz vsega tega lahko sklepamo da =>

Celotna ploščina te funkcija, ki jo omejuje x-os

je enaka $2\cdot S_2 = 2\cdot\frac{3\pi}{2} = 3\pi$

⇓

Iz tega lahko sklepamo, da je osnovna

funkcija oziroma gausova krivulja $f(x) = \frac{1}{x^2+1}$ =>

ima ploščino $\int_{-\infty}^{\infty} \frac{1}{x^2+1}\,dx = \pi$ => Gausov

integral !

Domača naloga => 2 list (OR)

1) Izračunaj =>

$$\int \left(x\sqrt{x} - \frac{3}{x^2}\right) dx = \frac{2}{5} x^{\frac{5}{2}} + 3x^{-1} + C$$

2) Izračunaj =>

$$\int \frac{(1-x)^2}{x}\, dx = \int \frac{1-2x+x^2}{x}\, dx = \ln|x| - 2x + \frac{x^2}{2} + C$$

3) Določi družino funkcij, katerih odvod je enak $f'(x) = \frac{(x^2-2)^2}{x^3}$

$$F(x) = \int \frac{(x^2-2)^2}{x^3}\, dx = \int \frac{x^4-4x^2+4}{x^3}\, dx = \frac{x^2}{2} - 4\ln|x| - 2x^{-2} + C$$

3) Določi družino funkcij, katerih odvod je enak $f'(x) = \frac{(x^2-2)^2}{x^3}$

$$F(x) = \int \frac{(x^2-2)^2}{x^3}\,dx = \int \frac{x^4-4x^2+4}{x^3}\,dx = \frac{x^2}{2} - 4\ln|x| - 2x^{-2} + C$$

4) Izračunaj =>

$$\int \frac{2-\sin^3 x}{\sin^2 x}\,dx = -2\cot x + \cos x + C = \cos x\left(-2\sin^{-1}x + 1\right) + C$$

5) Izračunaj točno vrednost določenega integrala =>

$$\int_0^{\frac{\pi}{2}} (\sin x + 3\cos x - x)\,dx = \left(3\sin x - \cos x - \frac{x^2}{2}\right)\Big|_0^{\frac{\pi}{2}} = \left(3 - \frac{\pi^2}{8}\right) - (-1) = 4 - \frac{\pi^2}{8}$$

6) Nariši graf funkcije $f(x)$ in izračunaj ploščino lika med grafom $f(x)$ in osjo x na intervalu $[1, 2]$

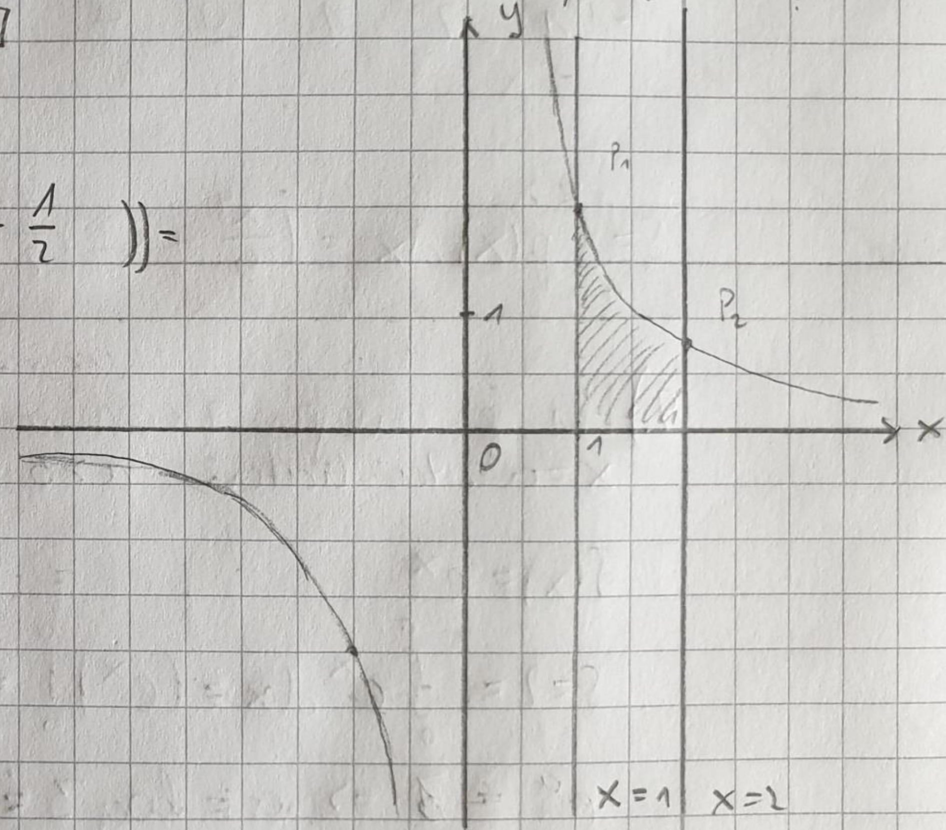
$$\int_0^{\frac{\pi}{2}} (\sin x + 3\cos x - x)\,dx = \left(3\sin x - \cos x - \frac{x^2}{2}\right)\Big|_0^{\frac{\pi}{2}} = \left(3 - \frac{\pi^2}{8}\right) - (-1) = 4 - \frac{\pi^2}{8}$$

6.) Nariši graf funkcije $f(x)$ in izračunaj ploščino lika med grafom $f(x)$ in osjo x na intervalu $[1, 2]$

$$f(x) = \frac{2}{x^3}$$

$$S = 2\int_1^2 x^{-3}\,dx = 2\left(-\frac{1}{2}x^{-2}\right)\Big|_1^2 = 2\left(\left(-\frac{1}{8}\right) - \left(-\frac{1}{2}\right)\right) =$$

$$S = \frac{3}{4}$$



7.) Dan je polinom $p(x)$. Izračunaj ploščino lika, ki ga omejujeta abscisna (x-os) in graf polinoma.

$p(x) = x^3 - 3x - 2 \Rightarrow p'(x) = 3x^2 - 3 = 0$

$x = \pm 1$ $T_1(1, -4) \Rightarrow$ Min

$y = -4$ $T_2(-1, 0) \Rightarrow$ Max

$y = 0$

⇓

Ničle = naše meje

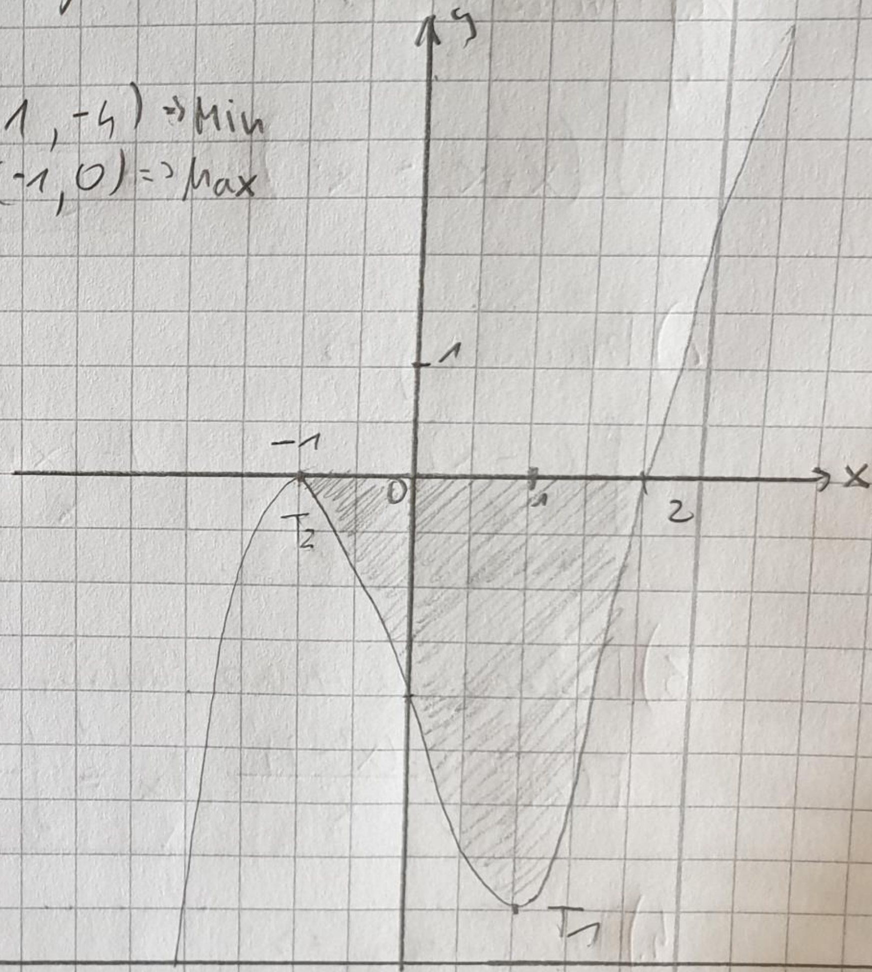
	1	0	-3	-2
		-1	1	2
-1	1	-1	-2	0

$\Rightarrow x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x_1 = -1$ (S) $\Rightarrow$ Max

$x_2 = 2$ (L)



$S = -\int_{-1}^{2} p(x)\,dx = -\left(\frac{x^4}{4} - \frac{3x^2}{2} - 2x\right)\Big|_{-1}^{2} =$

$S = -\left((-6) - \left(\frac{3}{4}\right)\right) =$

$S = +6\frac{3}{4}$

8.) Dan je polinom p(x), izračunajte ploščino lika, ki ga omejujeta obe koordinatni osi in graf polinoma.

$p(x) = x^3 + 3x^2 + 3x + 1 \Rightarrow 3x^2 + 6x + 3 = 0$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1 \quad y = 0$

$T(-1, 0)$

Ni ekstrema ampak prevojna točka!

Ničle ⇒

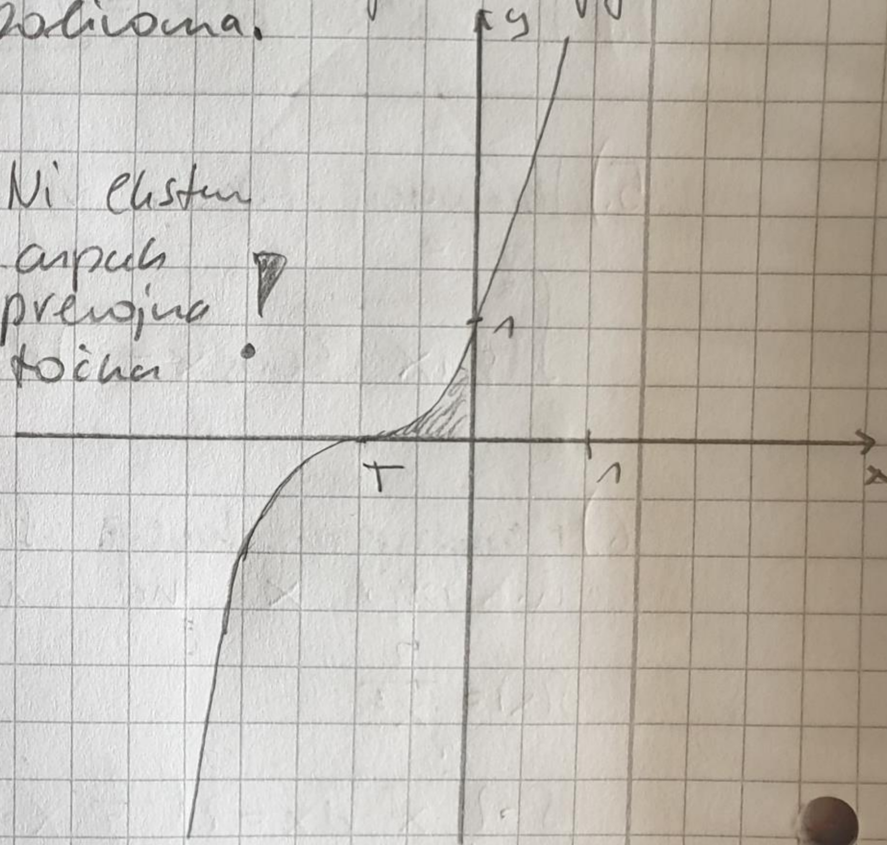
	1	3	3	1
		-1	-2	-1
-1	1	2	1	0

$\Rightarrow x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x_{1,2,3} = -1$

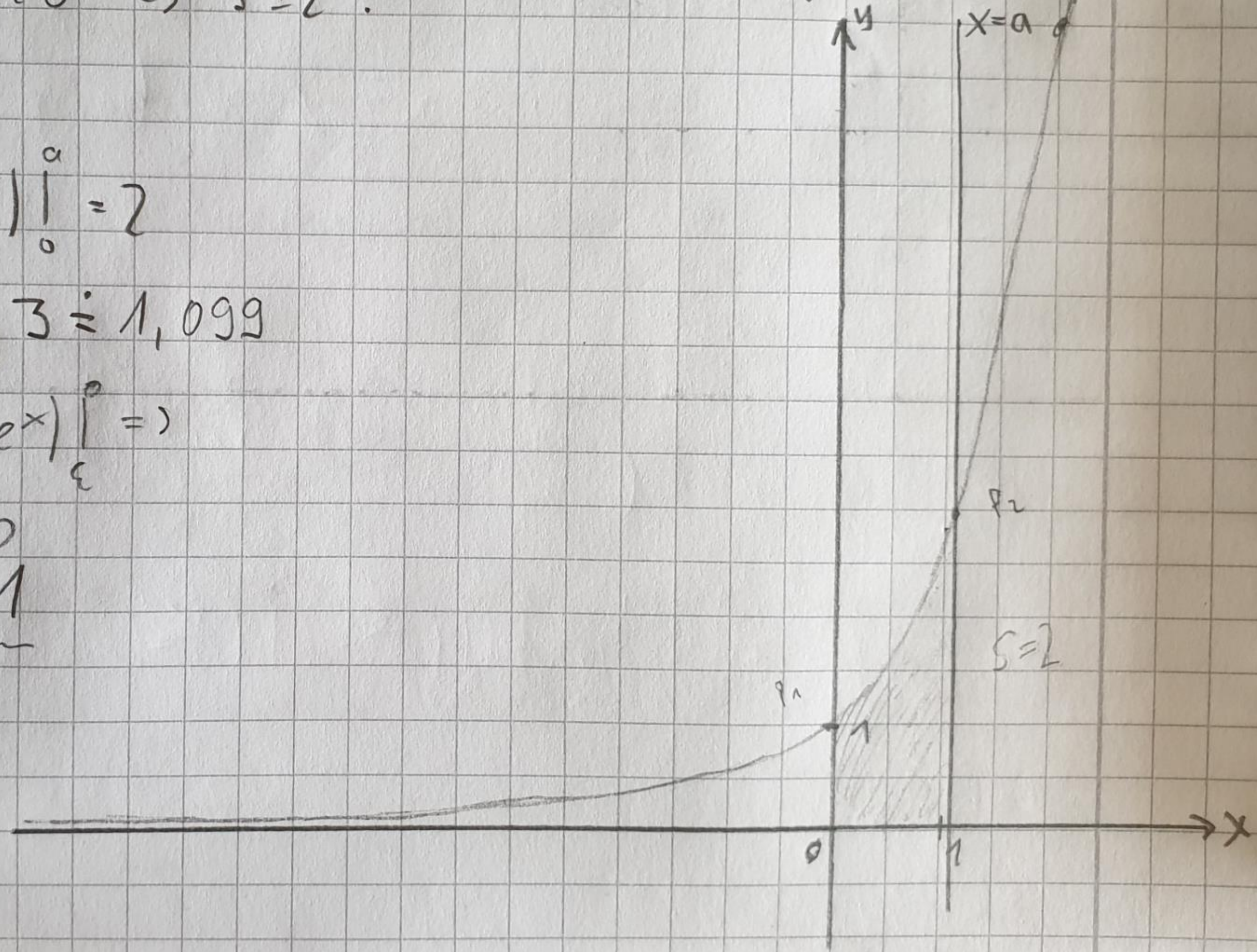
$\hookrightarrow (x+1)^3 = 0$



$S = \int_{-1}^{0} p(x)\,dx = \left(\frac{x^4}{4} + x^3 + \frac{3}{2}x^2 + x\right)\Big|_{-1}^{0} = \frac{1}{4}$

9.) Lik je omejen z ...

9) Lik je omejen z olnena koor. osema, grafom funkcije f(x) in premico $x = a$. Za kateri $a \geq 0 \Rightarrow S = 2$?



$f(x) = e^x$

$S = 2 = \int_0^a e^x\,dx = (e^x)\Big|_0^a = 2$

$e^a = 3 \Rightarrow a = \log 3 \doteq 1{,}099$

$S = \int_{-\infty}^0 e^x\,dx = \lim_{\varepsilon \to -\infty} (e^x)\Big|_\varepsilon^0 \Rightarrow$

$S \Rightarrow \lim_{\varepsilon \to -\infty} (e^0 - e^\varepsilon) = 1$ ($e^\varepsilon \Rightarrow 0$)

10.) Realno število a je pozitivno. Graf funkcije f(x) in koordinatni osi omejijo lik s ploščino 12. Izračunaj število a.

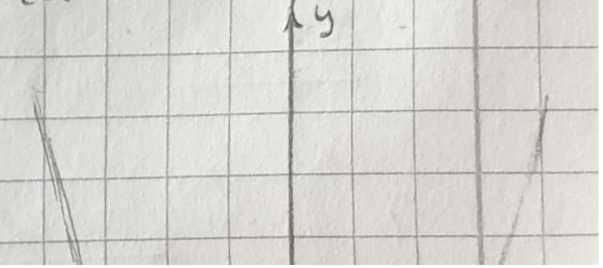
$f(x) = -x^3 + a \Rightarrow$ Ničla $= \sqrt[3]{a} = x$ (Naša meja)

$S = 12 = \int_0^{\sqrt[3]{a}} f(x)\,dx = \left(-\frac{x^4}{4} + ax\right)\Big|_0^{\sqrt[3]{a}} =$

$S \Rightarrow a\sqrt[3]{a} - a\sqrt[3]{a}\,\frac{1}{4} = 12 \Rightarrow 3a\sqrt[3]{a} = 48$

$a^{4/3} = 16 \Rightarrow a = 8$

11.) Izračunaj ploščino lika, ki ga omejita premica in parabola.



$f(x) = x^2 - 1 = (x-1)(x+1) \Rightarrow T(0,-1)$

$y = 1 \Rightarrow 2 = x^2 \Rightarrow x = \pm\sqrt{2}$

$S \Rightarrow a\sqrt[3]{a} - a\sqrt[3]{a}\,\frac{1}{4} = 12 \Rightarrow 3a\sqrt[3]{a} = 48$

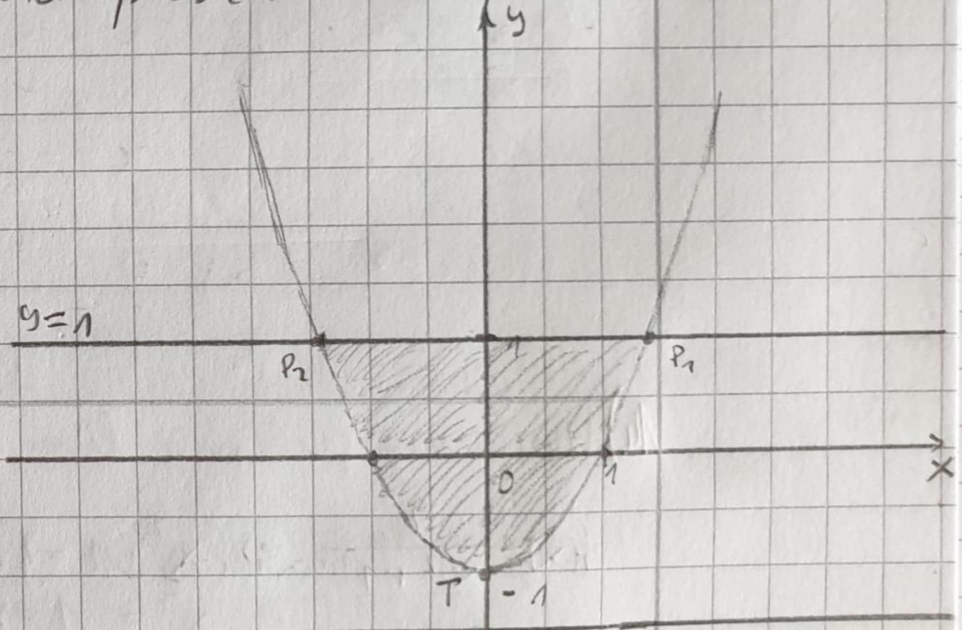
$a^{4/3} = 16 \Rightarrow \boxed{a = 8}$

11.) Izračunaj ploščino lika, ki ga oklepata premica in parabola.

$f(x) = x^2 - 1 = (x-1)(x+1) \Rightarrow T(0, -1)$

$y = 1 \Rightarrow 2 = x^2 \Rightarrow x = \pm\sqrt{2}$

Presečišča $\Rightarrow P_1(\sqrt{2}, 1)\ P_2(-\sqrt{2}, 1)$



$S = \int_{-\sqrt{2}}^{\sqrt{2}} (2 - x^2)\,dx = \left(2x - \frac{x^3}{3}\right)\Big|_{-\sqrt{2}}^{\sqrt{2}}$

$S = \left(2\sqrt{2} - \frac{2\sqrt{2}}{3}\right) - \left(-2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) = \frac{8\sqrt{2}}{3}$

12.) Dani sta funkciji f(x) in g(x) izračunaj ploščino lika, ki ga oklepata.

12) Dani sta funkciji f(x) in g(x) izračunaj ploščino lika, ki ga omejijeta.

$f(x) = -x^2 + 6x - 5 = -(x-5)(x-1) = -(x-3)^2 + 4$ $\quad T(3,4)$

$g(x) = -x + 5$

Presečišče => $f(x) = g(x)$

$x^2 - 7x + 10 = 0$

$(x-5)(x-2) = 0$

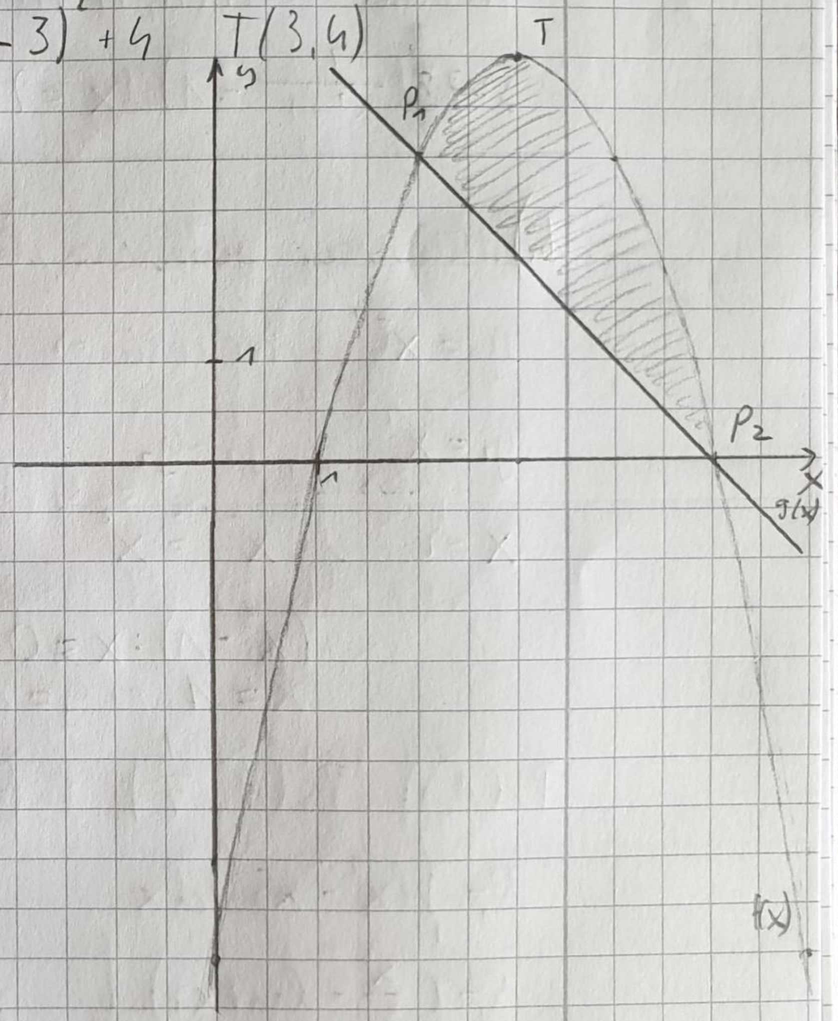
$x = 5 \quad y = 0 \quad P_2(5,0)$

$x = 2 \quad y = 3 \quad P_1(2,3)$

$S = \int_2^5 (-x^2 + 7x - 10)\,dx =$

$S = \left(-\frac{x^3}{3} + \frac{7}{2}x^2 - 10x\right)\Big|_2^5 =$

$S = \left(-4\frac{1}{6}\right) - \left(-8\frac{2}{3}\right) = 4\frac{1}{2}$



13.) Nariši grafa funkcij $f(x)$ in $g(x)$ ter izračunaj ploščino, ki jo oklepata

$f(x) = -x^2 + 1 \Rightarrow -(x-1)(x+1) \Rightarrow T_1(0,1)$

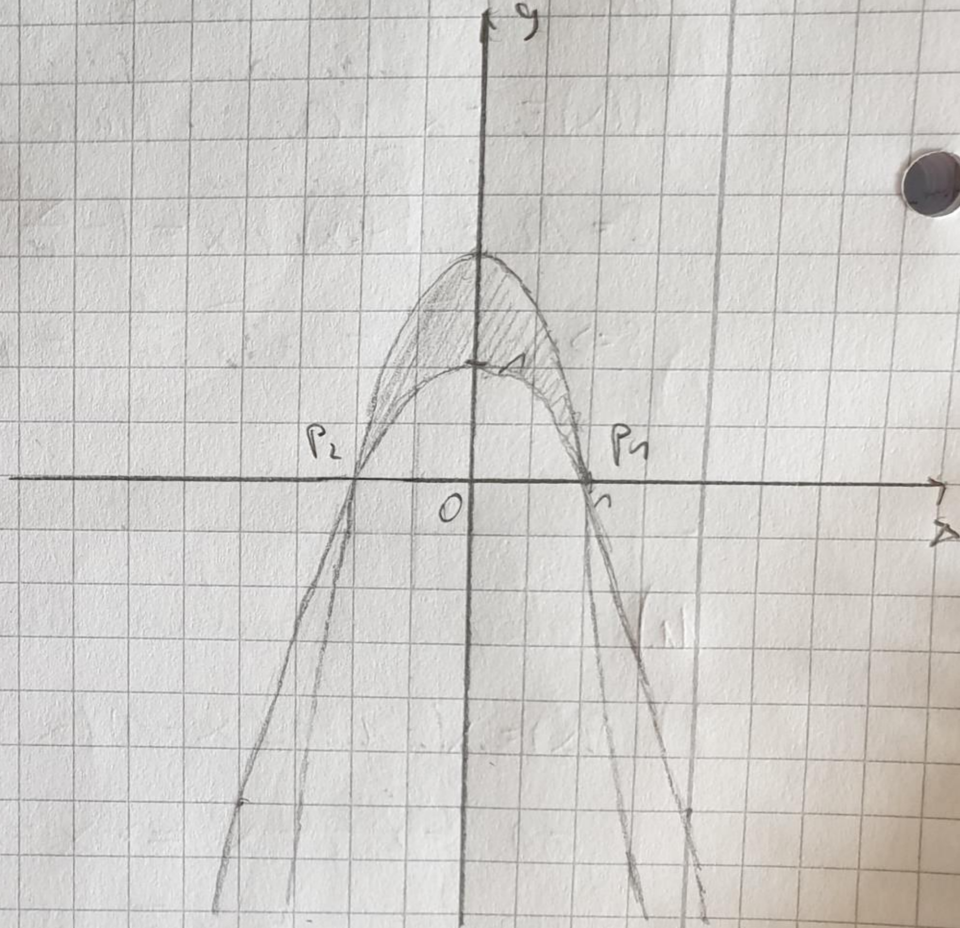
$g(x) = -2x^2 + 2 \Rightarrow -2(x-1)(x+1) \Rightarrow T_2(0,2)$

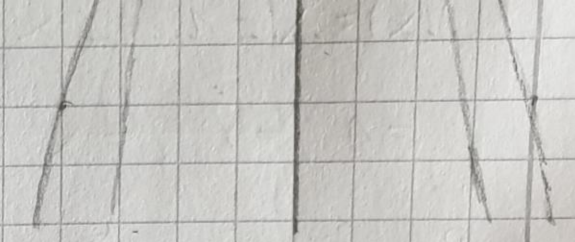
Presečišče $\Rightarrow f(x) = g(x)$

$x^2 = 1 \Rightarrow x = \pm 1 \quad P_1(1,0)$

$y = 0 \quad P_2(-1,0)$

$S = 2\int_0^1 (-x^2+1)\,dx = 2\left(-\frac{x^3}{3} + x\right)\Big|_0^1 = \frac{4}{3}$





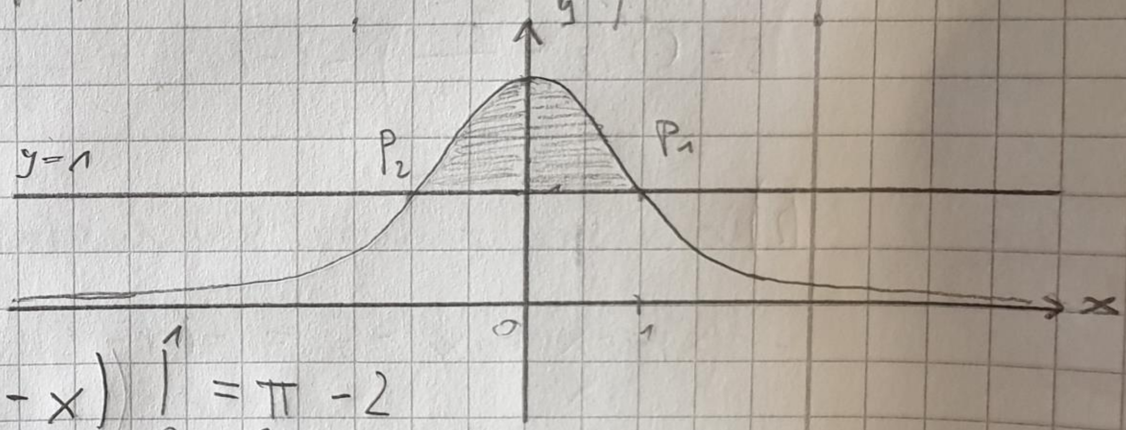
14.) Izračunaj ploščino lika, ki ga omejuje premica $y=1$ in krivulja $f(x)$

$f(x)=\frac{2}{x^2+1}$ $\Rightarrow$ $x^2+1=2$

$x^2=1 \Rightarrow x=\pm 1$

$y=1$ $\qquad$ $y=1$

Presečišče $\Rightarrow$ $P_1(1,1)$ $P_2(-1,1)$

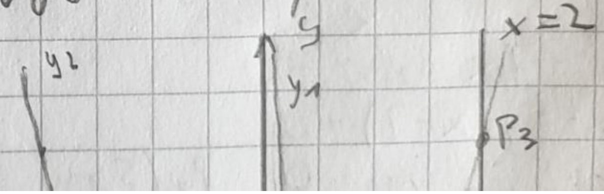


$$S=2\int_0^1\left(\frac{2}{x^2+1}-1\right)dx = 2\left(2(\arctan x)-x\right)\Big|_0^1 = \pi-2$$

15.) Izračunaj ploščino lika, ki ga omejujejo krivulje in premice $\Rightarrow$

$y_1=x^{-1}$ | Presečišča $\Rightarrow$

$y_2 = x^2$ ...



15.) Izračunaj ploščino lika, ki ga omejijo krivulje in premice $\Rightarrow$

$$\left.\begin{array}{l} y_1 = x^{-1} \\ y_2 = x^2 \\ x = 2 \end{array}\right\}$$

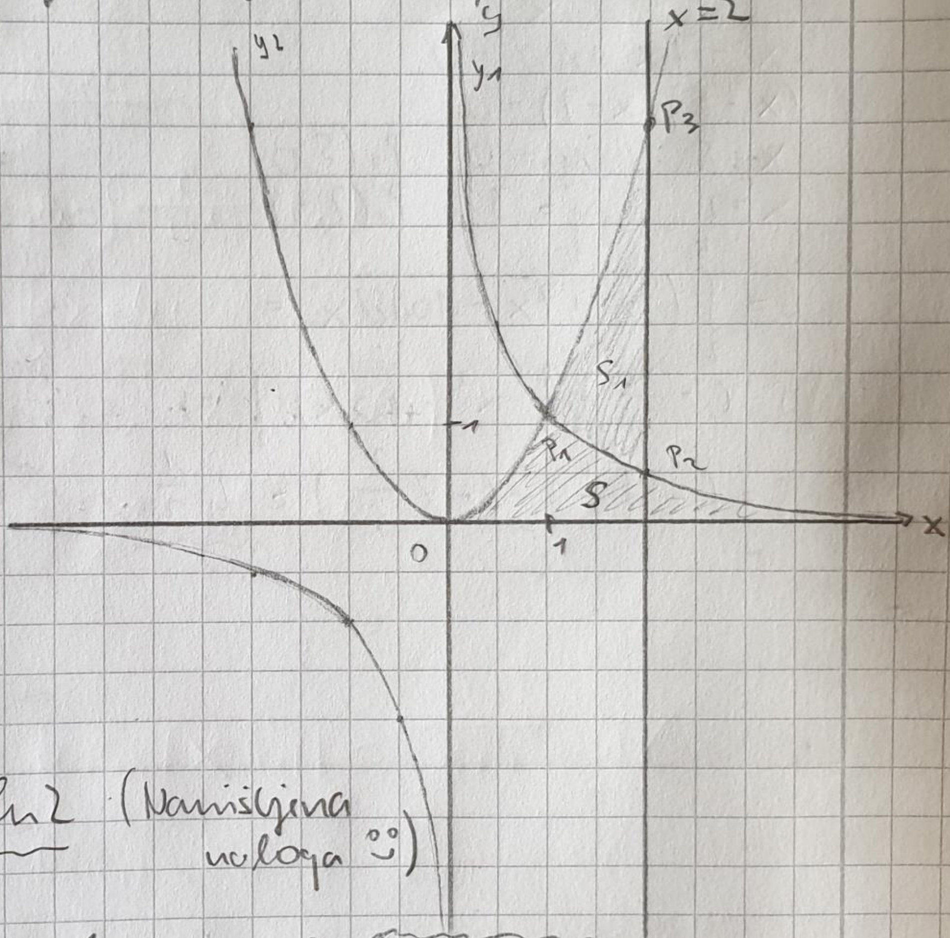
Presečišča $\Rightarrow$

$y_1 = y_2$

$x^{-1} = x^2$

$(x^3 - 1) : x = 0$

$x = 1 \Rightarrow y = 1$



$P_1(1,1) \quad P_2(2, \frac{1}{2}) \quad P_3(-2,4)$

$S_1 = \int_1^2 (x^2 - x^{-1})\,dx$

$S_1 = \left(\frac{x^3}{3} - \ln|x|\right)\Big|_1^2$

$S_1 = \left(\frac{8}{3} - \ln 2\right) - \left(\frac{1}{3}\right) = \frac{7}{3} - \ln 2$ (Namišljena naloga :))

Naš S $\Rightarrow$

$S = \int_0^1 (x^2)\,dx + \int_1^2 (x^{-1})\,dx = \left(\frac{x^3}{3}\right)\Big|_0^1 + \ln|x|\,\Big|_1^2 = \frac{1}{3} + \ln 2$

Domača Naloga => 3 list (VR)

1.) Dan je polinom $p(x)$

a.) Izračunajte ničle, ekstrem, prevoj in ga narišite.

b.) Poiščite presečišče s premico $x+y-4=0$

c.) Izračunajte ploščino med njima na intervalu kjer je maksimum.

$p(x) = -x^3+6x^2-9x+4$ =>

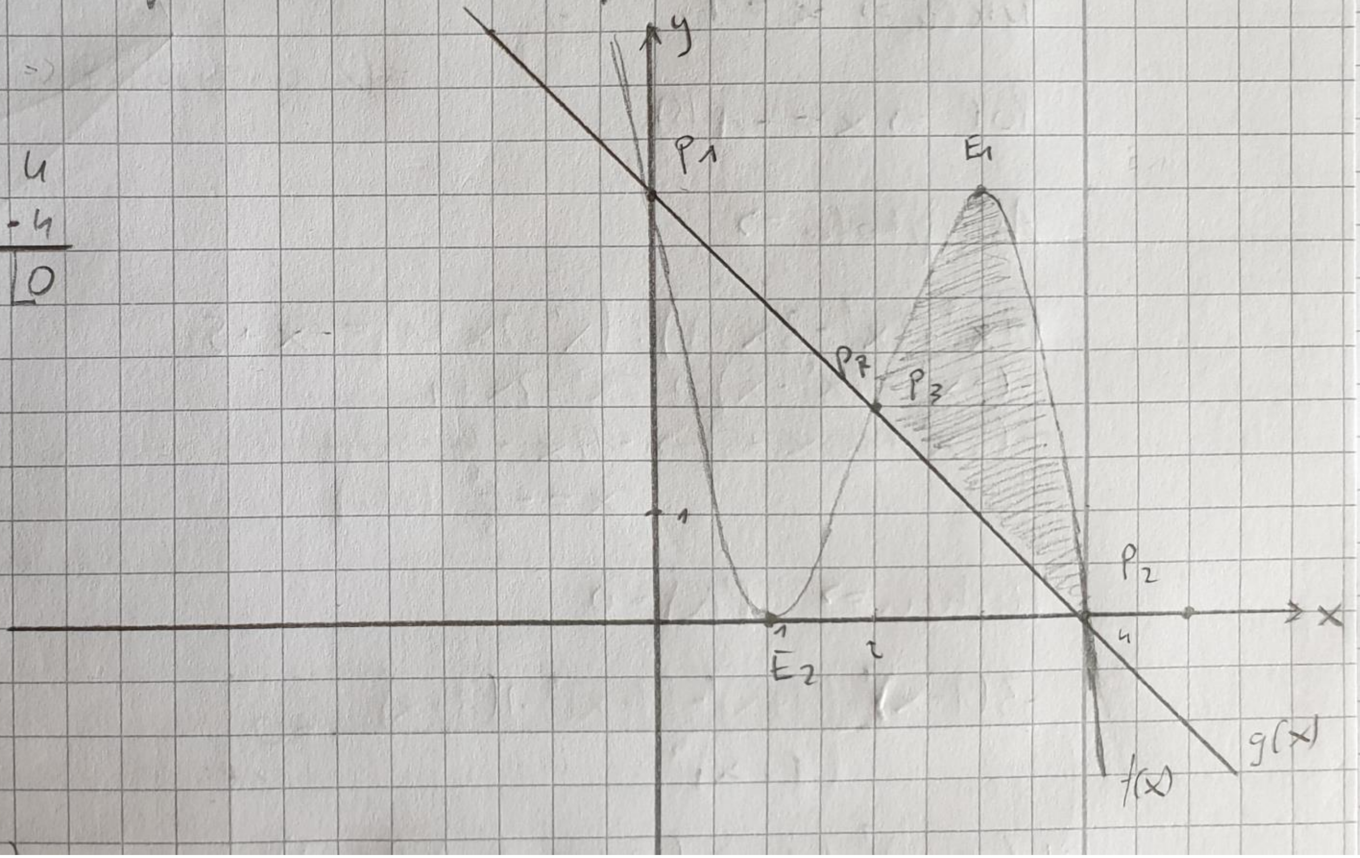
Ničle =>

	-1	6	-9	4
		-1	5	-4
1	-1	5	-4	0

$x^2-5x+4=0$

$(x-4)(x-1)=0$

N => $x_{1,2}=1$ (S) $x=4$



Ekstrem => $p'(x)=0$

$-3x^2+12x-9=0$

$x^2-4x+3=0$

$(x-3)(x-1)=0$

Ekstrem => $p'(x)=0$

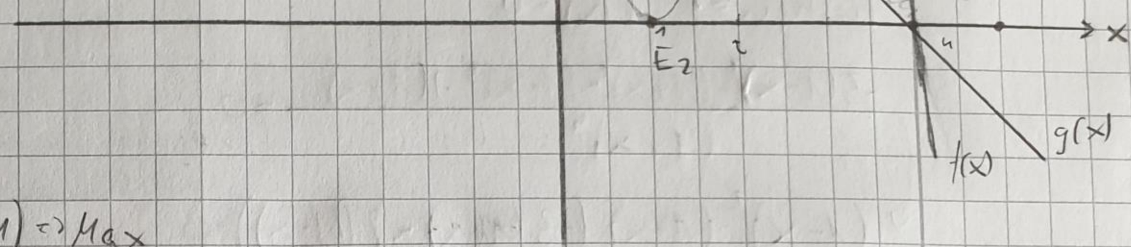
$-3x^2+12x-9=0$

$x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3 \quad y= \qquad E_1(3,4) \Rightarrow Max$

$x=1 \quad y=0 \qquad E_2(1,0) \Rightarrow Min$



Prevoj => $p''(x)=0$

$-6x+12=0 \Rightarrow x=2 \quad P_2(2,2)$

$y=2$

$\lim_{x\to\infty} f(x) = -\infty \qquad \lim_{x\to-\infty} f(x)=\infty$

$g(x)=4-x$

Max => konkavna

$-$ E_1 $+$ =>

Presečišče => $f(x)=g(x)$

$x^3-6x^2+9x-4+4-x=0$

$x^3-6x^2+8x=0$

$x(x^2-6x+8)=0$

$x(x-4)(x-2)=0$

$x=0 \quad y=4 \quad P_1(0,4)$

$x=4 \quad y=0 \quad P_2(4,0)$

$x=2 \quad y=2 \quad P_3(2,2)$

↳ $S=\int_2^4(-x^3+6x^2-8x)\,dx = \left(-\frac{x^4}{4}+2x^3-4x^2\right)\Big|_2^4$

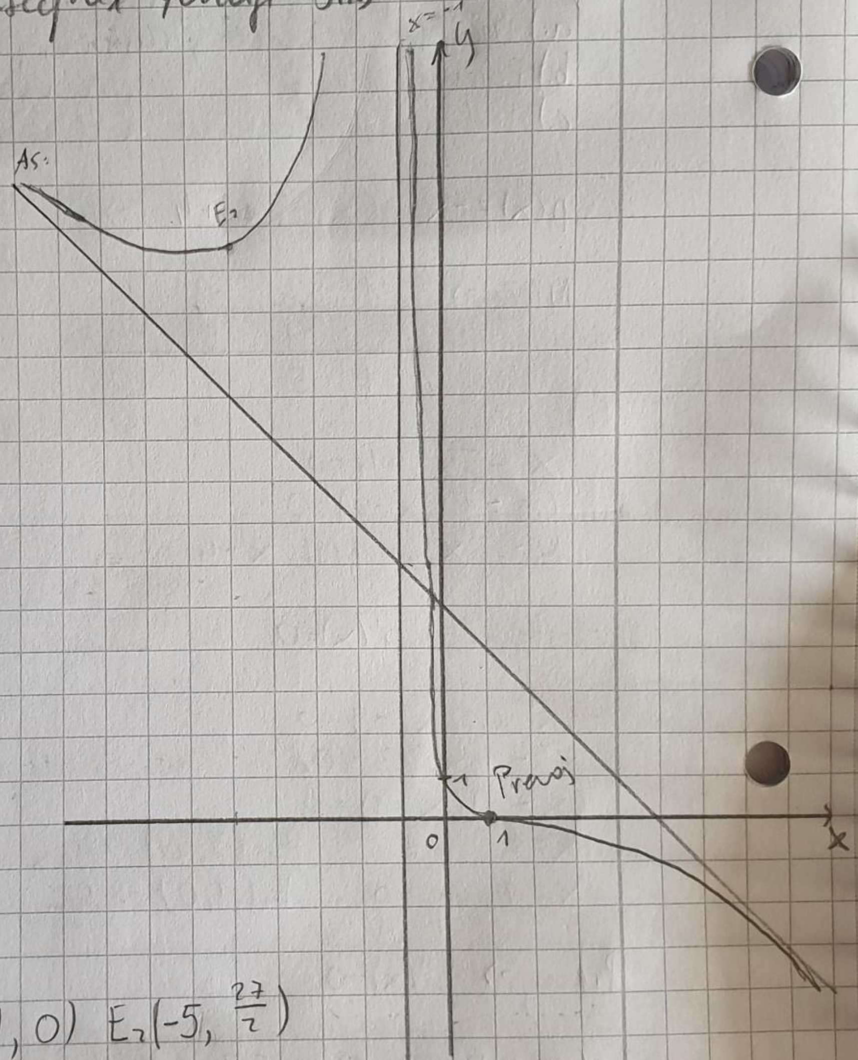
$S=(0)-(-4)=4$

2.) Dana je funkcija $f(x)$

a) Izračunajte ničle in pole, asimptoto, ekstrem, prevoj in jo narišite

b) Izračunajte še nedoločeni integral funkcije $f(x)$

$$f(x) = \frac{(1-x)^3}{(1+x)^2}$$



Ničla $\Rightarrow x = 1$

Pol $\Rightarrow x = -1$ (S)

Asimptota $\Rightarrow$

$(-x^3+3x^2-3x+1) : (x^2+2x+1) = -x+5$

$-(-x^3-2x^2-x) \Rightarrow 5x^2-2x+1$

$-(5x^2+10x+5) \rightarrow -12x-4 \Rightarrow$ ost

$x = -\frac{1}{3}$ (sečna)

Ekstrem $\Rightarrow f'(x) = 0$

$$\frac{-3(1-x)^2\cdot(1+x)^2-(1-x)^3\cdot 2(1+x)}{(1+x)^4} = 0$$

$(1-x)^2\cdot(1+x)\left(-3(1+x)-2(1-x)\right) = 0$

$(1-x)^2(1+x)(-3-3x-2+2x) = 0$

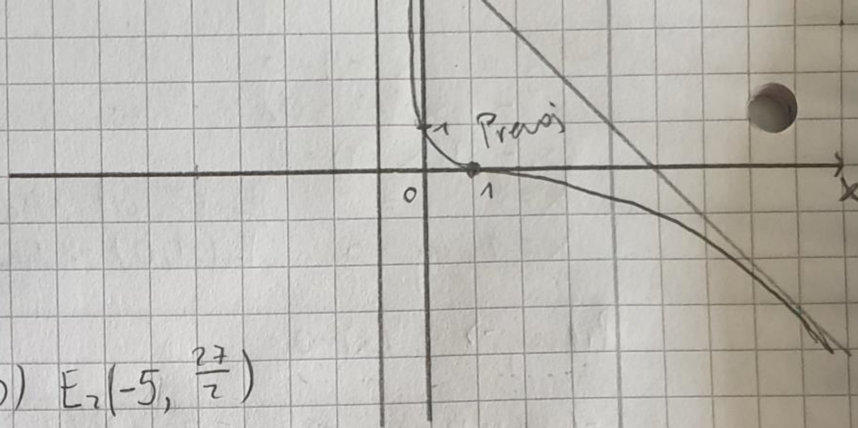
$(1-x)^2(1+x)(-5-x) = 0 \qquad E_1(1,0) \quad E_2\left(-5, \frac{27}{2}\right)$

$$\frac{-3(1-x)^2\cdot(1+x)^2-(1-x)^3\cdot 2(1+x)}{(1+x)^4}=0$$

$$(1-x)^2\cdot(1+x)\left(-3(1+x)-2(1-x)\right)=0$$

$$(1-x)^2(1+x)\left(-3-3x-2+2x\right)=0$$

$$(1-x)^2(1+x)\left(-5-x\right)=0 \qquad E_1(1,0) \quad E_2\left(-5,\frac{27}{2}\right)$$



E_1: $-$ (1,0) $-$; E_1 => prevoj funkcije

E_2: $-$ $\left(-5,\frac{27}{2}\right)$ $+$; E_2 => ∪ => minimum (konveksnost)

$$\int f(x)\,dx = \int(-x+5)\,dx - \int\frac{12x+4}{(x+1)^2}\,dx$$

$$-4\int\frac{3x+1}{(x+1)^2}\,dx = -4\left(\int\frac{3u}{u^2}\,du - \int\frac{2}{u^2}\,du\right)$$

$x+1=t \Rightarrow dt=du$

$$-4\left(\int 3u^{-1}\,du - 2\int u^{-2}\,du\right)$$

$$= -12\ln|u| - 8u^{-1} + C$$

$-$ $(\ldots\frac{1}{2})$ $+$ $\Rightarrow$ ∪ $\Rightarrow$ minimum (konveksnost)

E_1

$$\int f(x)\,dx = \int(-x+5)\,dx - \int \frac{12x+4}{(x+1)^2}\,dx$$

$$-4\int\frac{3x+1}{(x+1)^2}\,dx = -4\left(\int\frac{3u}{u^2}\,du - \int\frac{2}{u^2}\,du\right)$$

$x+1 = t \Rightarrow dt = du$

$$-4\left(\int 3u^{-1}\,du - 2\int u^{-2}\,du\right)$$

$$= -12\ln|u| - 8u^{-1} + C$$

Rešitev $\Rightarrow$

$$\int f(x)\,dx = 5x - \frac{x^2}{2} - 12\ln|x+1| - \frac{8}{x+1} + C$$

Poenostavljeno $\Rightarrow$ $\frac{x(10-x)}{2} - 12\ln|x+1| - \frac{8}{x+1} + C$

3.) Dana je funkcija $f(x)$

a.) Izračunajte ničlo, pol, asimptoto, ekstrem in jo narišite.

b.) Zapišite enačbo tangente v točki $T(-2, 90)$

c.) Izračunajte $\int_{2}^{4} f(x)\,dx$

$$f(x) = \frac{x}{(x^2+4x-5)}$$

Ničla $\Rightarrow x=0$

Pola $\Rightarrow (x+5)(x-1)=0$

$x=-5 \quad x=1$

As $\Rightarrow y=0$

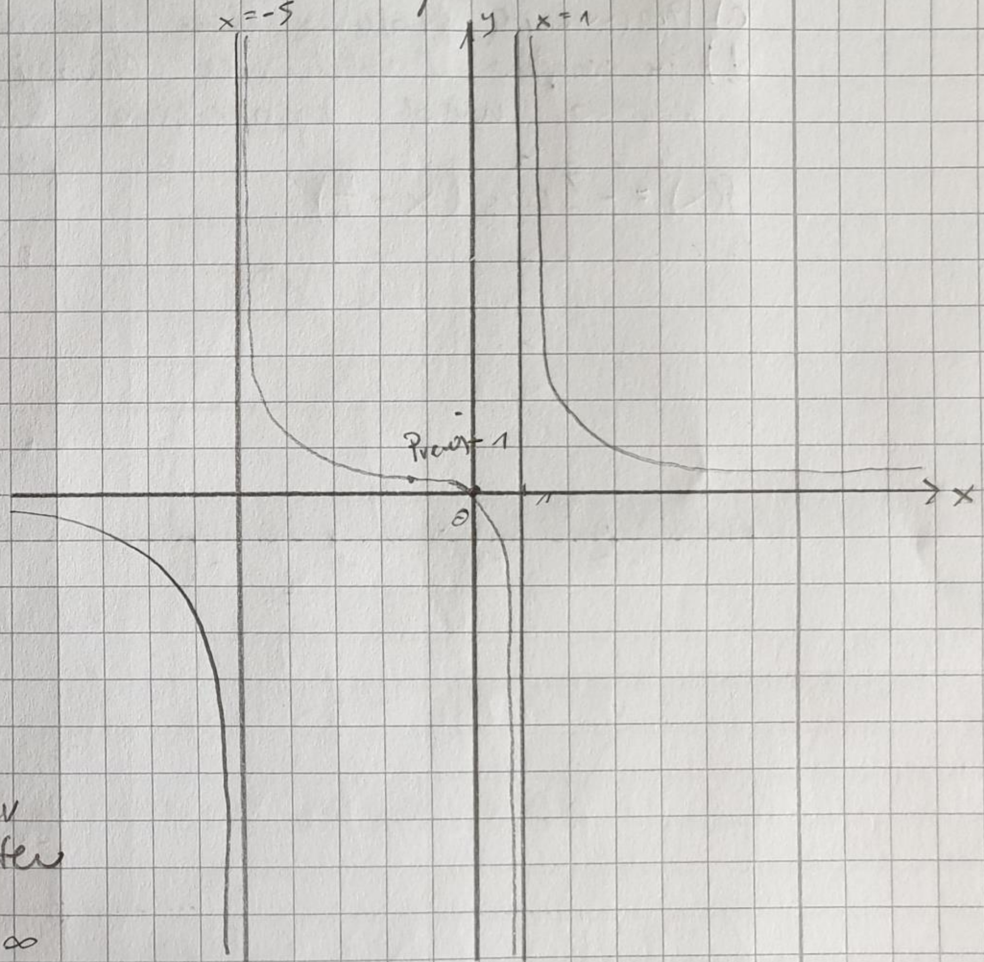
Ekstrem $\Rightarrow f'(x)=0$

$$\frac{x^2+4x-5-x(2x+4)}{(x^2+4x-5)^2}=0$$

$x^2+4x-5-2x^2-4x=0$

$x^2=-5 \Rightarrow$ Ni ekstrema, ker ni realnih rešitev

$\lim_{x\to\infty} f(x) = +$ $\quad \lim_{x\to-\infty} f(x) = -\infty$



$\lim_{x \to \infty} f(x) = +$ $\lim_{x \to -\infty} f(x) = -\infty$

Prevoj => $f''(x) = 0$

① odvod => $\frac{-(x^2+5)}{((x+5)(x-1))^2} \Rightarrow \frac{-(x^2+5)}{(x^2+4x-5)^2}$

② odvod => $\frac{-2x(x^2+4x-5)^2 + (x^2+5) \cdot 2 \cdot (x^2+4x-5) \cdot (2x+4)}{(x^2+4x-5)^4} = 0$

$-2x(x^4+8x^3-10x^2-40x+16x^2-25)+ \ldots\ldots$ Th napej

Zun priče $x \doteq -1{,}22$ $y = 0{,}14$

Enačba tangente =>

$T(-2, \frac{2}{9})$

$k = -\frac{1}{9}$

$y = -\frac{x}{9}$

Integral => $\int \frac{x}{x^2+4x-5}\,dx$

$\frac{A}{(x+5)} + \frac{B}{(x-1)} = \frac{x}{x^2+4x-5}$

$Ax - A + Bx + 5B = x$

Enačba tangente =)

$T(-2, \frac{2}{9})$

$k = -\frac{1}{9}$

$y = -\frac{x}{9}$

Integral =) $\int \frac{x}{x^2+4x-5}\,dx$

$$\frac{A}{(x+5)} + \frac{B}{(x-1)} = \frac{x}{x^2+4x-5}$$

$$Ax - A + Bx + 5B = x$$

① $A = 5B \Rightarrow 5B + B = 1 \Rightarrow B = \frac{1}{6} \Rightarrow A = \frac{5}{6}$

$$\frac{5}{6}\int \frac{1}{x+5}\,dx + \frac{1}{6}\int \frac{1}{x-1}\,dx$$

$$= \frac{5}{6}\ln|x+5| + \frac{1}{6}\ln|x-1| + C$$

Meje =) $S = \left(\frac{5}{6}\ln|x+5| + \frac{1}{6}\ln|x-1|\right)\Big|_2^4$

$S = \left(\frac{5}{6}\ln 9 + \frac{1}{6}\ln 3\right) - \left(\frac{5}{6}\ln 7 + \frac{1}{6}\ln 1\right)$

$S = \left(\frac{1}{6}\cdot 5\cdot 2\cdot \ln 3 + \frac{1}{6}\ln 3\right) - \frac{5}{6}\ln 7 = \frac{1}{6}(11\ln 3 - 5\ln 7)$

4) Dana je funkcija $f(x)$

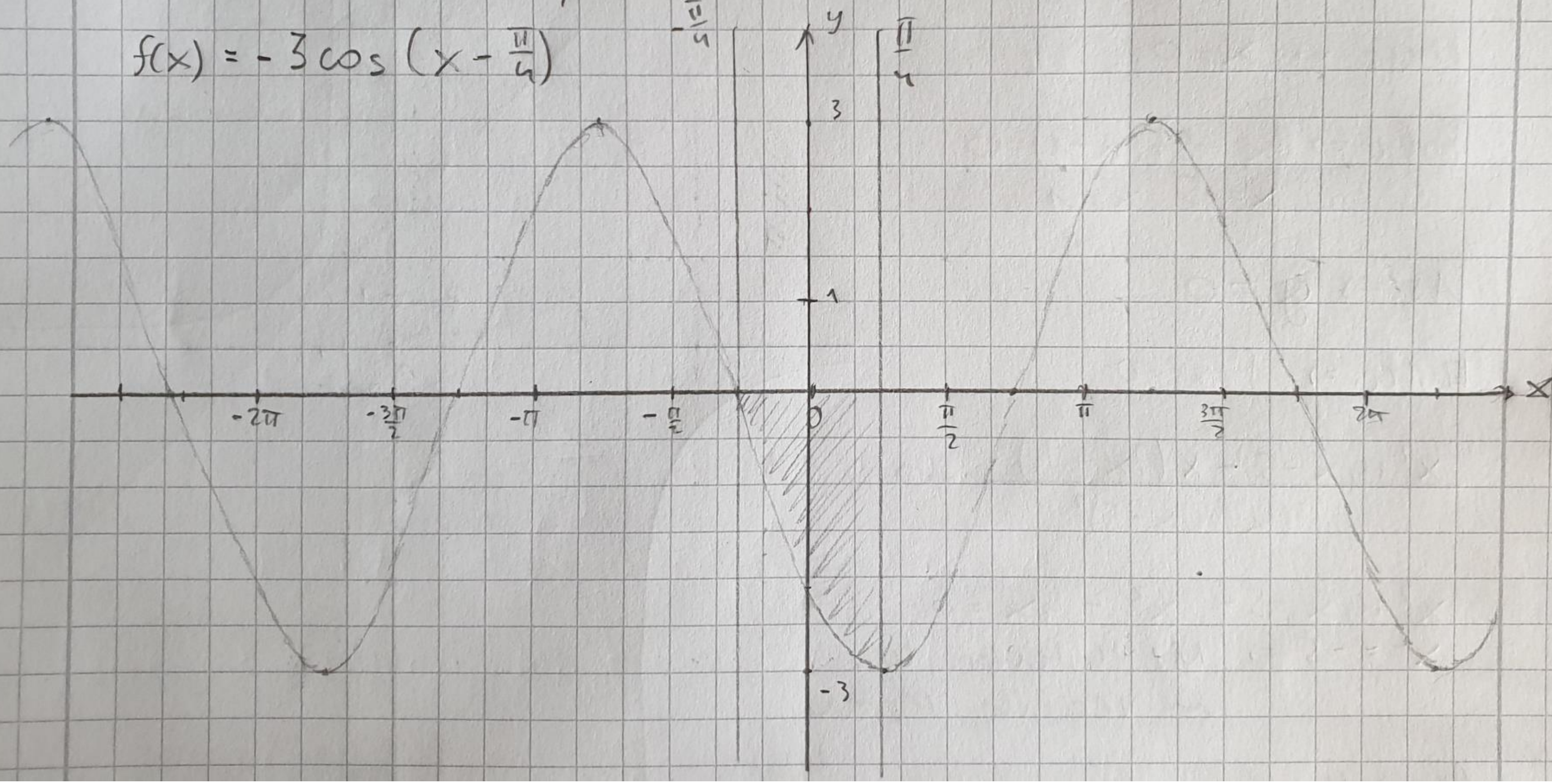
a.) Narišite jo in izračunajte neenačbo $f(x) < 0$

b.) Pod kolikšnim kotom seka ordinato os?

c.) Izračunajte ploščino lika na intervalu $[-\frac{\pi}{4}, \frac{\pi}{4}]$

d.) Izračunajte V vrtenine, če območje med x-osjo in grafom zavrtimo med zaporednima ničlama.

$$f(x) = -3\cos\left(x - \frac{\pi}{4}\right)$$



Ekstrem => $f'(x)=0$ => $3\sin(x-\frac{\pi}{4})=0$ $\quad t_1 = k\pi$ => $x_1 = \frac{\pi}{4} + k\pi$

$\sin(x-\frac{\pi}{4})=0$ $\quad t_2 = \pi + k\pi$ => $x_2 = \frac{5\pi}{4} + k\pi$

$x-\frac{\pi}{4}=t$ $\quad \sin t = 0$ =>

$x = \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$

Nicla => $-3\cos(x-\frac{\pi}{4})=0$

$\cos t = 0$ => $t_1 = \frac{\pi}{2} + k\pi$ => $x = \frac{3\pi}{4} + k\pi$

$f(0) = -\frac{3\sqrt{2}}{2}$ => $k = 3\sin(0 - \frac{\pi}{4}) = -\frac{3\sqrt{2}}{2}$ => $\alpha = 90 + \arctan k = 25{,}239°$

$= 25°14'$

$S = -\int_{-\pi/4}^{\pi/4} -3\cos(x-\frac{\pi}{4})\,dx = -(-3\sin(x-\frac{\pi}{4}))\Big|_{-\pi/4}^{\pi/4} = (0) - (3) = 3$

$u \to du = dx$

Volumen =>

$\pi \int_{\frac{3\pi}{4}=a}^{\frac{7\pi}{4}=b} (-3\cos(x-\frac{\pi}{4}))^2\,dx = 9\pi \int_a^b \cos^2(x-\frac{\pi}{4})\,dx$

[illegible]

$u \to du = dx$

Volumen =>

$$\pi \int_{\frac{3\pi}{4}=a}^{\frac{7\pi}{4}=b} \left(-3\cos\left(x-\frac{\pi}{4}\right)\right)^2 dx = 9\pi \int_a^b \cos^2\left(x-\frac{\pi}{4}\right) dx$$

$$\int \cos^2 x \, dx = \int 1 \, dx - \int \sin^2 x \, dx$$

Neenačba => $f(x) < 0$

$\Rightarrow \cos\left(x-\frac{\pi}{4}\right) = 0$

$\cos t < 0$

$t = \frac{\pi}{2} + k\pi$

$x = \frac{3\pi}{4} + k\pi$

⇓

$x \in \left(-\frac{\pi}{4} + 2k\pi \, ; \, \frac{3\pi}{4} + 2k\pi\right)$

↳ Graf + usi Max in Min !

$$= \int 1 dx - \frac{1}{2}\int (1 - \cos 2x) dx$$

$$= \int 1 dx - \frac{1}{2}\int 1 dx - \frac{1}{2}\int \cos 2x \, dx$$

$$= x - \frac{x}{2} - \frac{1}{4}\sin 2x + C$$

$$\Rightarrow 9\pi \left(\frac{x - \frac{\pi}{4}}{2} - \frac{1}{4}\sin\left(2x - \frac{\pi}{2}\right) \right) \Bigg|_{\frac{3\pi}{4}}^{\frac{7\pi}{4}}$$

$$V = 9\pi \left(\frac{3\pi}{4} - \frac{\pi}{4}\right) = 9\pi \cdot \frac{\pi}{2}$$

$$= \boxed{\frac{9\pi^2}{2}}$$

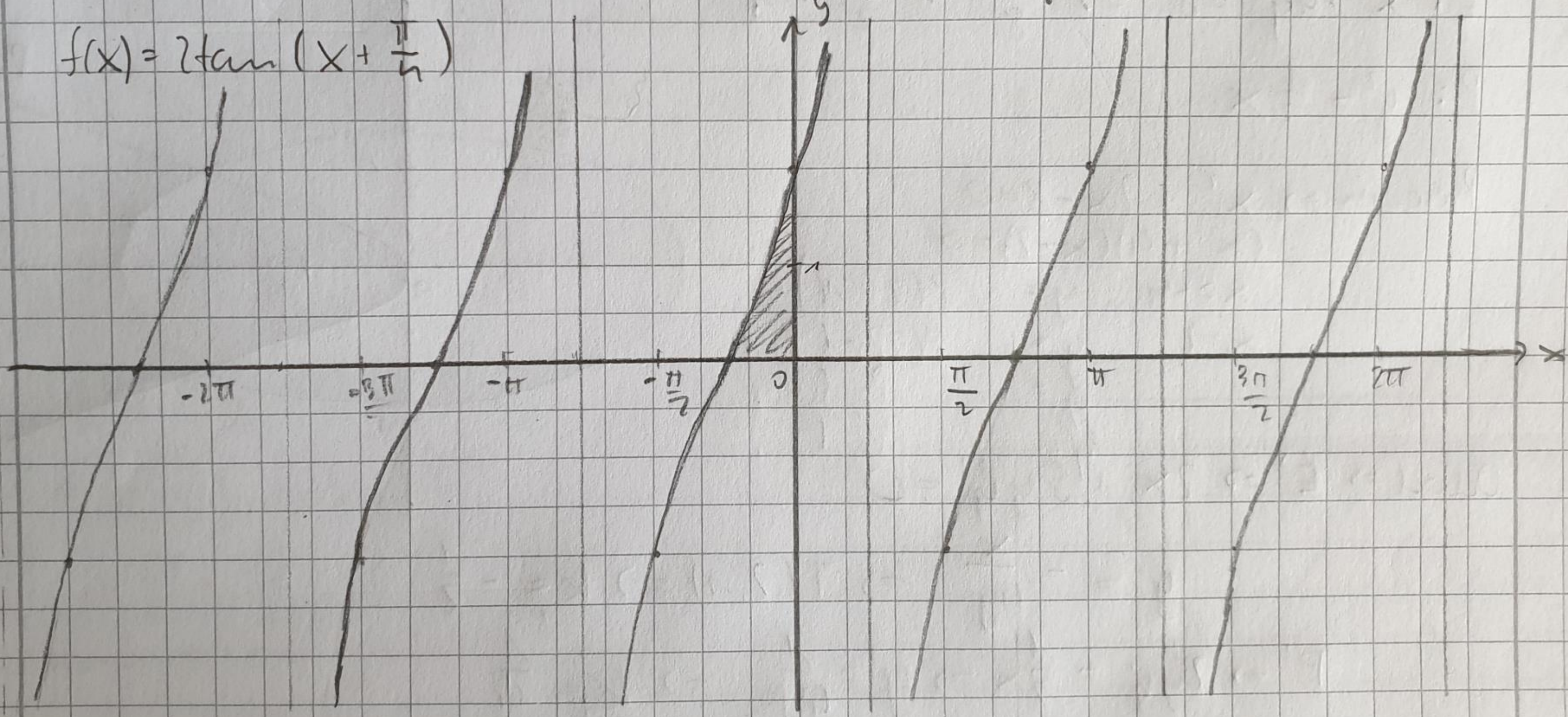
5.) Narišite graf funkcije $f(x)$

a.) Rešite neenačbo $f(x) \leq -2$

b.) Izračunajte kot in ostali presečišči, ki ga seka s $y = \frac{2\sqrt{3}}{3}$

c.) Izračunajte ploščino lika, ki ga oklepa graf in koor. osi.

$f(x) = 2\tan\left(x + \frac{\pi}{4}\right)$



Ničle $\Rightarrow$ $\tan\left(x + \frac{\pi}{4}\right) = 0$ $\quad z = k\pi \Rightarrow x = -\frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$

Ničle $\Rightarrow$ $\tan\left(x+\frac{\pi}{4}\right)=0$ $\quad z=k\pi \Rightarrow x=-\frac{\pi}{4}+k\pi \quad k\in\mathbb{Z}$

$\tan z = 0 \Rightarrow$

Poli $\Rightarrow$ $x+\frac{\pi}{4}=\frac{\pi}{2} \Rightarrow x=\frac{\pi}{4}+k\pi$

Neenačba $\Rightarrow$ $\cancel{2}\tan\left(x+\frac{\pi}{4}\right)\leq \cancel{-2} \Rightarrow \tan\left(x+\frac{\pi}{4}\right)\leq -1$

$x\leq -\frac{\pi}{2}+k\pi \Rightarrow x\leq \frac{\pi}{2}+k\pi$

Upoštevajoč se pole $\Rightarrow$

$x\in\left(\frac{\pi}{4}+k\pi\,;\,\frac{\pi}{2}+k\pi\right) \quad k\in\mathbb{Z}$

$\Downarrow$

Odvod $\Rightarrow$ $f'(x)=\frac{2}{\cos^2\left(x+\frac{\pi}{4}\right)}$

Presečišče $\Rightarrow$ $2\tan\left(x+\frac{\pi}{4}\right)=\frac{2\sqrt{3}}{3} \Rightarrow \tan\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{3} \Rightarrow x=-\frac{\pi}{12}+k\pi$

Kot $\quad f'\left(-\frac{\pi}{12}\right)=2\frac{2}{3} \Rightarrow \alpha=\arctan k = 69{,}44^\circ \doteq 69^\circ 27'$

$x \neq -\frac{\pi}{2} + k\pi \Rightarrow x \neq \frac{\pi}{2} + k\pi$

Upoštevajoč se pole $\Rightarrow$ $x \in \left(\frac{\pi}{4} + k\pi ; \frac{\pi}{2} + k\pi\right) \quad k \in \mathbb{Z}$

Odvod $\Rightarrow$ $f'(x) = \frac{2}{\cos^2(x+\frac{\pi}{4})}$

Presečišče $\Rightarrow$ $2\tan(x + \frac{\pi}{4}) = \frac{2\sqrt{3}}{3} \Rightarrow \tan(x+\frac{\pi}{4}) = \frac{\sqrt{3}}{3} \Rightarrow x = -\frac{\pi}{12} + k\pi$

kot $f'(-\frac{\pi}{12}) = 2\frac{2}{3} \Rightarrow \alpha = \arctan k = 69{,}44^\circ \doteq 69^\circ 27'$

Ploščina $\Rightarrow$

$$S = 2\int_{-\frac{\pi}{4}}^{0} \tan(x+\frac{\pi}{4})\,dx = 2(-\ln|\cos(x+\frac{\pi}{4})|)\Big|_{-\frac{\pi}{4}}^{0} = 2\cdot(-\ln\frac{\sqrt{2}}{2} \Rightarrow \frac{1}{\sqrt{2}} = (2)^{-\frac{1}{2}}$$

$$= 2\cdot(-(-\tfrac{1}{2})\ln 2)$$

$$= \ln 2$$

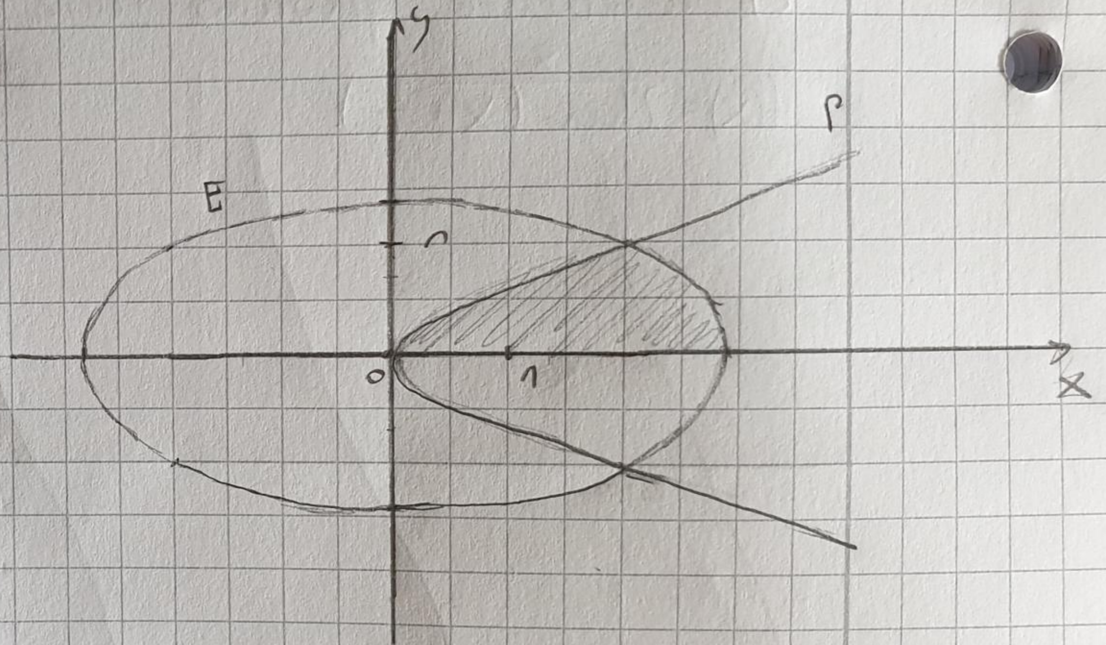
6.) Dani sta dve krivulji

a) Izračunajte njuni presečišči, kot pod katerim se sekata v I k. + graf

b) Izračunajte volumen telesa, ki nastane če v I k. to območje zavrtimo.

$E: x^2 + 4y^2 = 8 \Rightarrow \frac{x^2}{8} + \frac{y^2}{2} = 1$

$P: y^2 = \frac{1}{2}x$



Presečišče $\Rightarrow x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4 \quad y = \quad P(2, 1)$

$x = 2 \quad y = \quad P(2, -1)$

Odvod $\Rightarrow E' \Rightarrow 2x + 8yy' = 0$

$y' = -\frac{x}{4y} \Rightarrow T(2,1) \Rightarrow k_1 = -\frac{1}{2}$

$P' \Rightarrow 2yy' = \frac{1}{2}x \Rightarrow y' = \frac{1}{4y} \Rightarrow k_2 = \frac{1}{4}$

k.t $\Rightarrow \alpha = \arctan k_2 - \arctan k_1 = 40{,}6° = 40°, 36'$

$E: x^2 + 4y^2 = 8 \Rightarrow \quad 8 \quad 2$

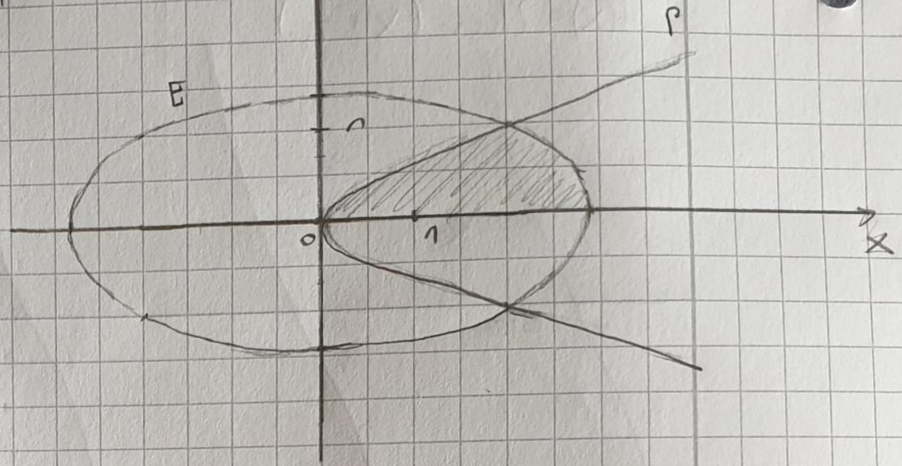
$P: \; y^2 = \frac{1}{2}x$

Presečnice $\Rightarrow x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4 \quad y = \quad P(2, 1)$

$x = 2 \quad y = \quad P(2, -1)$



Odvod $\Rightarrow E' \Rightarrow 2x + 8yy' = 0$

$y' = -\frac{x}{4y} \Rightarrow T(2,1) \Rightarrow k_1 = -\frac{1}{2}$

$P' \Rightarrow 2yy' = \frac{1}{2}x \Rightarrow y' = \frac{1}{4y} \Rightarrow k_2 = \frac{1}{4}$

Kot $\Rightarrow \alpha = \arctan k_2 - \arctan k_1 = 40{,}6° = 40°\,36'$

$$V = \pi \int_0^2 \left(\sqrt{\frac{x}{2}}\right)^2 dx + \pi \int_2^{2\sqrt{2}} \left(\sqrt{2 - \frac{x^2}{4}}\right)^2 dx$$

$$V = \pi \int_0^2 \frac{x}{2}\, dx + \pi \int_2^{2\sqrt{2}} \left(2 - \frac{x^2}{4}\right) dx$$

$$V = \frac{\pi}{2}\left(\frac{x^2}{2}\right)\Big|_0^2 + \pi\left(2x - \frac{x^3}{12}\right)\Big|_2^{2\sqrt{2}} = \pi + \frac{(10 + 8\sqrt{2})\pi}{3}$$

$$V = \frac{\pi}{3}\left(3 - 10 + 8\sqrt{2}\right) = \frac{\pi}{3}\left(8\sqrt{2} - 7\right)$$

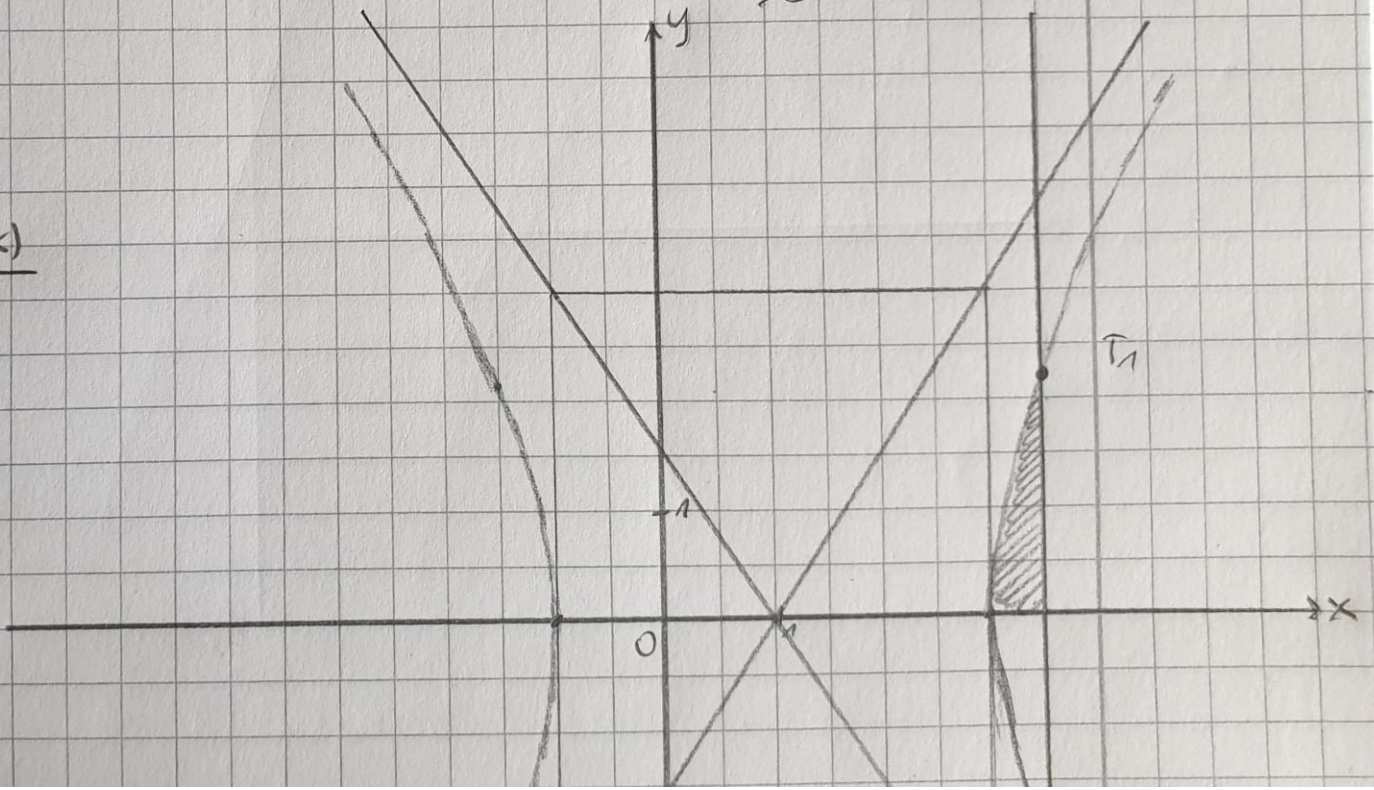
7.) Dana je hiperbola

a.) Najdite p in v točki $T(\frac{7}{2}; y \geq 0)$ zapišite enačbo tangente

b.) Izračunaj dolžino tetive, ki jo od premice $x = \frac{7}{2}$ odreže hiperbola

c.) Izračunajte volumen vrtenine, ki nastane če hiperbolo zavrtimo okoli abscise osi, ki ga omejijo p hiperbola, $x = \frac{7}{2}$ in $y = 0$

$$H: 9x^2 - 4y^2 - 18x - 27 = 0 \Rightarrow 9(x-1)^2 - 4y^2 = 36 = \frac{(x-1)^2}{4} - \frac{y^2}{9} = 1$$



Odvod $\Rightarrow$ $H' \Rightarrow$

$$H' \Rightarrow 18x - 8yy' - 18 = 0$$

$$y' = \frac{-18(1-x)}{8y} = -\frac{9(1-x)}{4y}$$

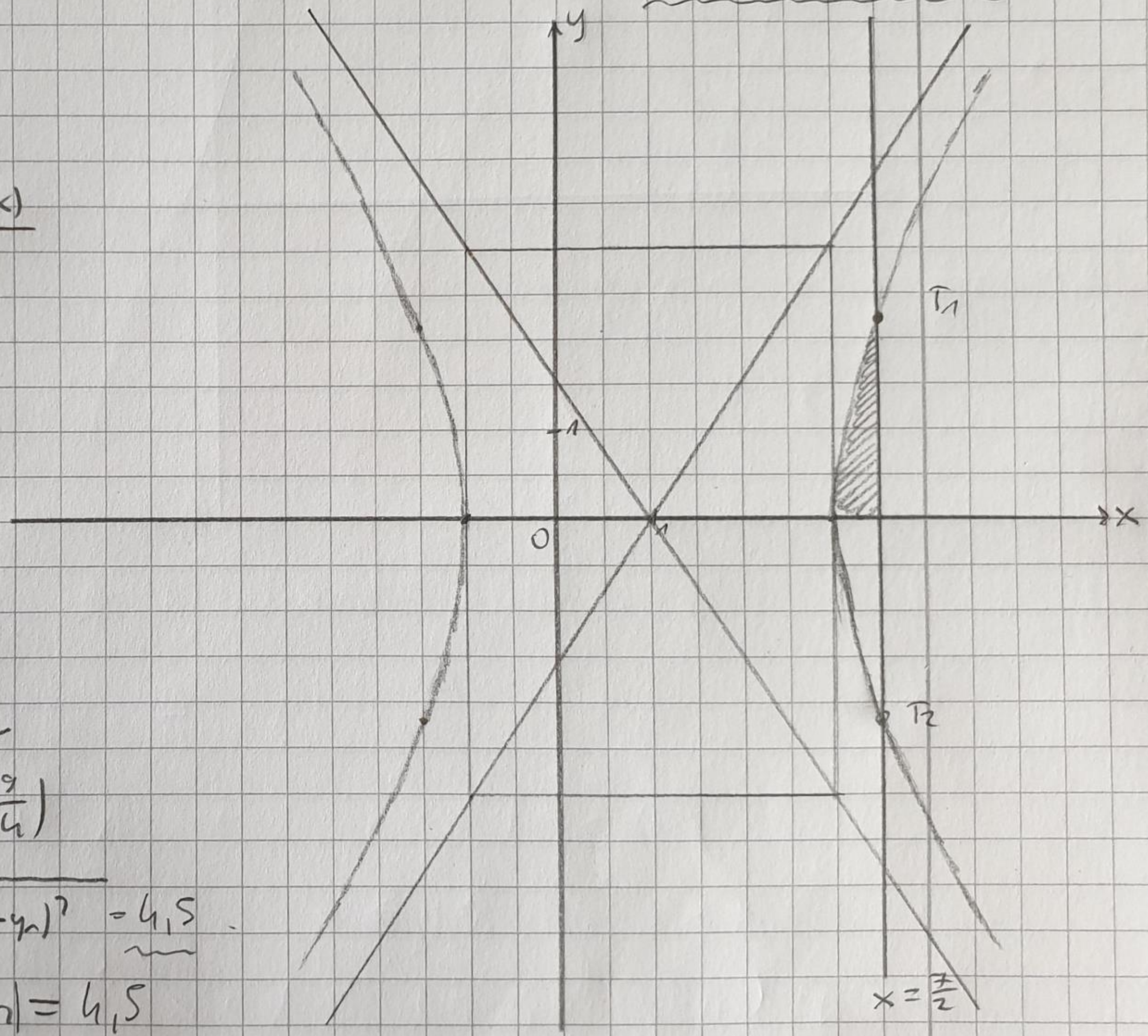
$$T\left(\frac{7}{2}, \frac{9}{4}\right) \Rightarrow k = \frac{5}{2}$$

$$y = \frac{5x}{2} - \frac{13}{2}$$

$$x = \frac{7}{2} \Rightarrow y = ?$$

[illegible] $x = \frac{7}{2}$ [illegible] $y = 0$

$H: 9x^2 - 4y^2 - 18x - 27 = 0 \Rightarrow 9(x-1)^2 - 4y^2 = 36 = \frac{(x-1)^2}{4} - \frac{y^2}{9} = 1$



Odod => $H' \Rightarrow$

$H' \Rightarrow 18x - 8yy' - 18 = 0$

$y' = \frac{-18(1-x)}{8y} = -\frac{9(1-x)}{4y}$

$T(\frac{7}{2}, \frac{9}{4}) \Rightarrow k = \frac{5}{2}$

$y = \frac{5x}{2} - \frac{13}{2}$

$x = \frac{7}{2} \Rightarrow y = ?$

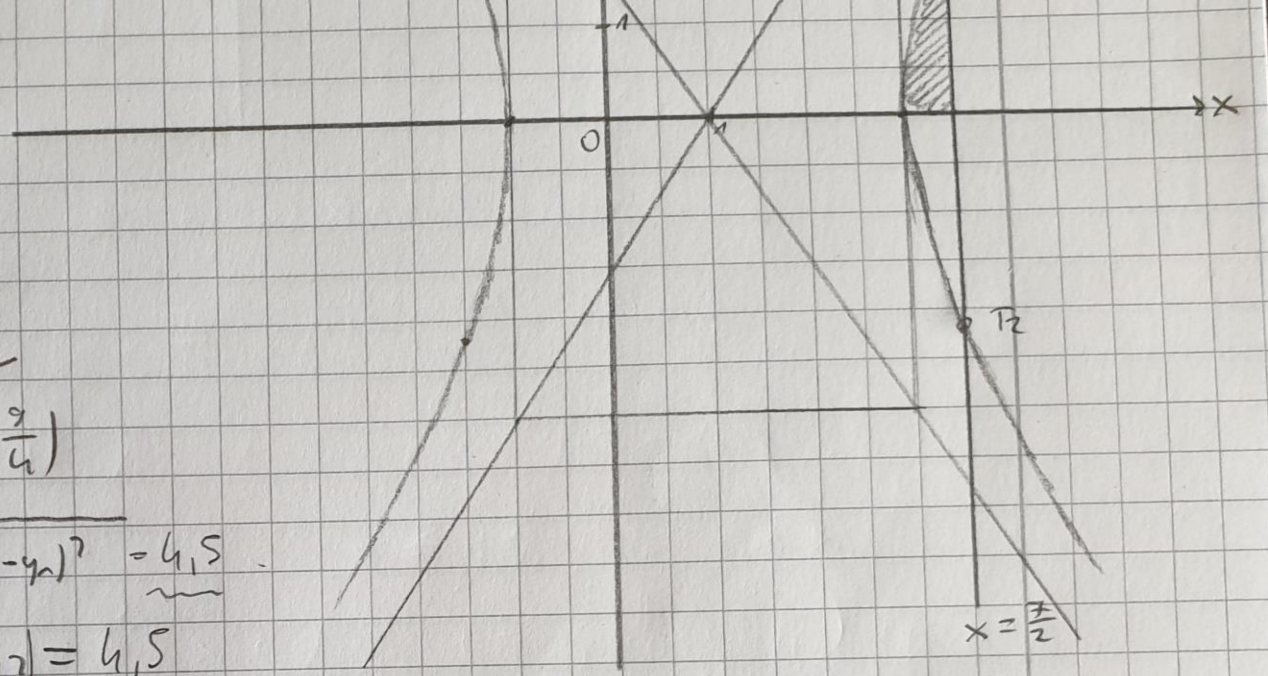
$y_1 = 2\frac{1}{4} \qquad y_2 = -2\frac{1}{4}$

$T_1(\frac{7}{2}, \frac{9}{4}) \quad T_2(\frac{7}{2}, -\frac{9}{4})$

① $d(T_1, T_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 4,5$

② lako [illegible] $\Rightarrow y_1 + |y_2| = 4,5$

$y = \frac{5x}{2} - \frac{15}{2}$



$x = \frac{7}{2} \Rightarrow y = ?$

$y_1 = 2\frac{1}{4} \qquad y_2 = -2\frac{1}{4}$

$T_1\left(\frac{7}{2}, \frac{9}{4}\right) \quad T_2\left(\frac{7}{2}, -\frac{9}{4}\right)$

① $d(T_1, T_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 4{,}5$

② lahko tudi $\Rightarrow y_1 + |y_2| = 4{,}5$

Volumen $\Rightarrow$

$$V = \pi \int_3^{7/2} \left(\sqrt{\frac{9x^2}{4} - \frac{9x}{2} - \frac{27}{4}}\right)^2 dx = \pi \int_3^{7/2} \left(\frac{9x^2}{4} - \frac{9x}{2} - \frac{27}{4}\right) dx$$

$$V = \frac{9\pi}{4}\left(\frac{x^3}{3} - x^2 - 3x\right)\Big|_3^{7/2} = \frac{9\pi}{4}\left(\left(-8\frac{11}{24}\right) - \left(-9\right)\right) = \frac{39\pi}{32}$$