MATURA 2002
(ON)

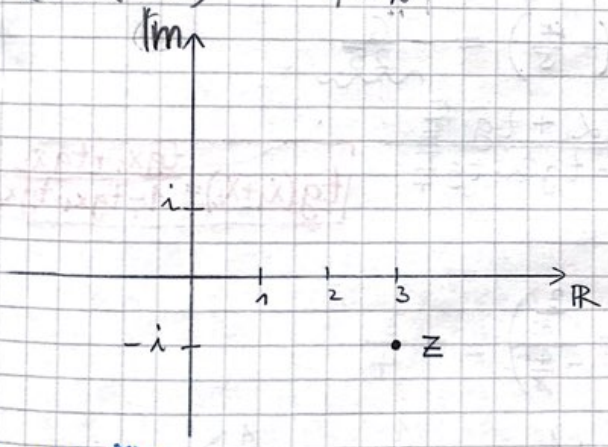
① S katerimi izmed števil 2, 3, 4, 5, 6 in 9 je deljivo število 10203040506 ? V tabeli obkrožite pravilne odgovore.

Št. 10203040506 je deljivo:

z 2	(DA)	NE
s 3	(DA)	NE
s 4	DA	(NE) → če je s 4 deljivo št., ki ga predstavljata zadnji dve števki
s 5	DA	(NE)
s 6	(DA)	NE → če je število deljivo z 2 in hkrati s 3
z 9	DA	(NE) → če je vsota števk deljiva z 9

② Zapišite število $z = \frac{5-5i}{2-i}$ v obliki a+bi, pri čemer sta a in b realni števili. V kompleksni ravnini narišite točko, ki predstavlja št. z.

$$z = \frac{(5-5i)(2+i)}{(2-i)(2+i)} = \frac{10-10i+5i-5i^2}{4-i^2} = \frac{15-5i}{5} = 3-i$$



③ Rešite enačbo:

$$\log_{8} 2\sqrt{2} = x$$

$$8^x = 2\sqrt{2}$$

$$2^{3x} = 2^1 \cdot 2^{\frac{1}{2}}$$

$$2^{3x} = 2^{\frac{3}{2}} \Rightarrow 3x = \frac{3}{2}$$

$$x = \frac{1}{2}$$

④ Rešite kvadratno neenačbo:

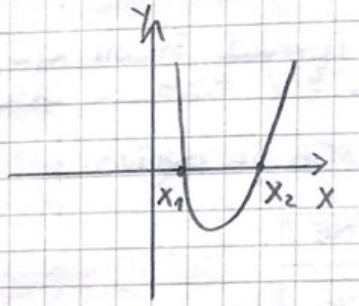
$\frac{x^2}{2} + x \leq \frac{3}{2} \quad / \cdot 2$

$x^2 + 2x - 3 \leq 0$

$(x+3)(x-1) \leq 0$

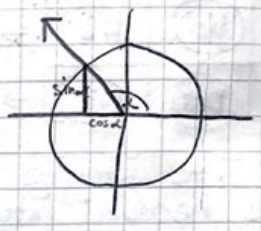
$x_1 = -3 \quad x_2 = 1$

$\mathbb{R}: (-3, 1)$



$\mathbb{R}: (x_1, x_2)$

⑤ Naj bo α topi kot $(\frac{\pi}{2} < \alpha < \pi)$ in $\sin\alpha = 3/5$. Izračunajte točni vrednosti $\sin 2\alpha$ in $\operatorname{tg}(\alpha + \pi/4)$.



$\rightarrow \sin 2\alpha = 2 \cdot \sin\alpha \cdot \cos\alpha$

$\sin^2\alpha + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - \sin^2\alpha$

$\cos^2\alpha = 1 - (3/5)^2 = \frac{25}{25} - \frac{9}{25} = \frac{16}{25}$

$\cos\alpha = \pm\sqrt{16/25} = -\frac{4}{5}$

$\sin 2\alpha = 2 \cdot \frac{3}{5} \cdot \left(-\frac{4}{5}\right) = -\frac{6}{25}$

$\rightarrow \operatorname{tg}(\alpha + \frac{\pi}{4}) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\frac{\pi}{4}}{1 - \operatorname{tg}\alpha \cdot \operatorname{tg}\frac{\pi}{4}}$

$\operatorname{tg}(x_1 \pm x_2) = \frac{\operatorname{tg}x_1 \pm \operatorname{tg}x_2}{1 \mp \operatorname{tg}x_1 \cdot \operatorname{tg}x_2}$!

$\operatorname{tg}\frac{\pi}{4} = 1$

$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$

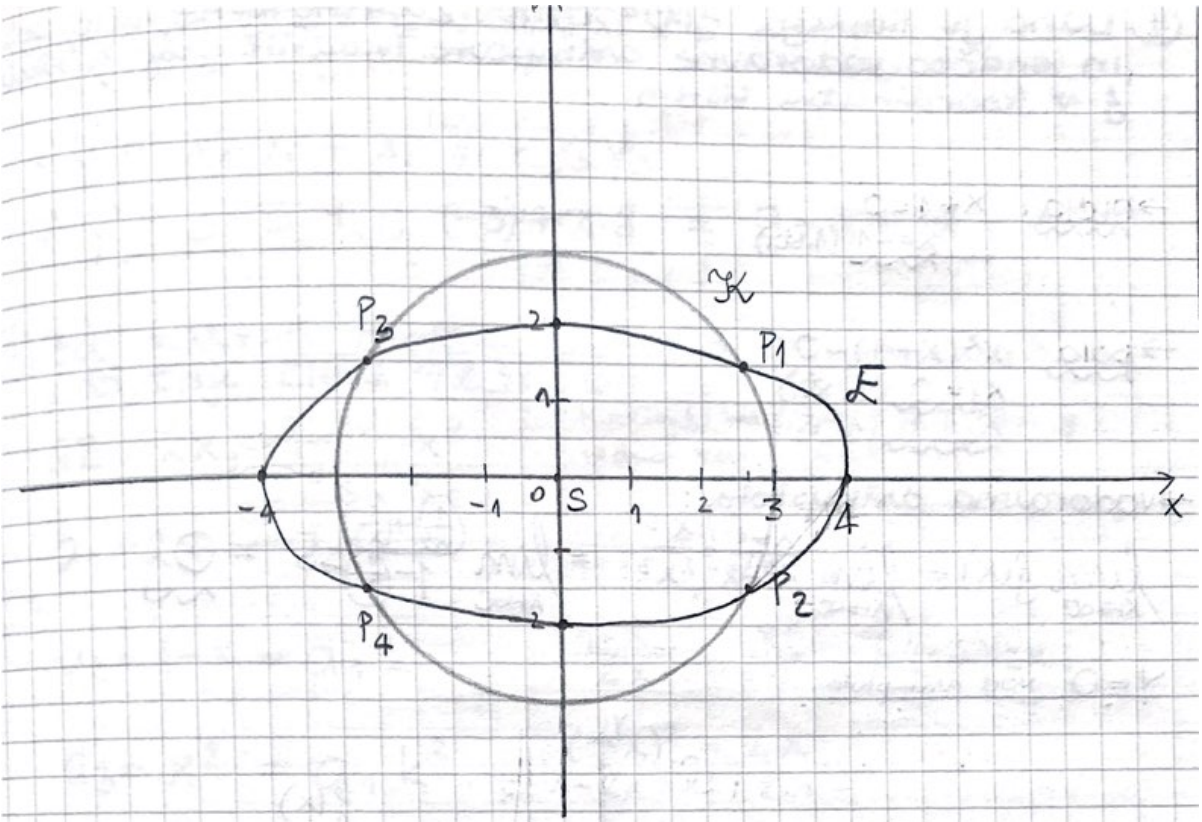
$\operatorname{tg}(\alpha + \frac{\pi}{4}) = \frac{-\frac{3}{4} + 1}{1 - (-\frac{3}{4} \cdot 1)} = \frac{\frac{1}{4}}{1 + \frac{3}{4}} = \frac{\frac{1}{4}}{\frac{7}{4}} = \frac{1}{7}$

⑥ V koordinatni sistem skicirajte krivulji z enačbama $x^2 + y^2 = 9$ in $x^2 + 4y^2 = 16$. Izračunajte točne koordinate presečišč teh dveh funkcij.

$K: x^2 + y^2 = 9 \; ; \; S(0,0), \; r = 3$

$E: x^2 + 4y^2 = 16 \quad / : 16$

$\frac{x^2}{16} + \frac{y^2}{4} = 1 \; ; \; S(0,0), \; a = 4, \; b = 2$



Presečišča funkcij:

$$\left.\begin{array}{l} x^2+y^2=9 \\ x^2+4y^2=16 \end{array}\right) -$$

$$-3y^2=-7 \quad /:(-3)$$

$$y^2=\frac{7}{3}$$

$$y=\pm\sqrt{\frac{7}{3}}$$

$$x^2=9-y^2$$

$$x^2=\frac{27}{3}-\frac{7}{3}$$

$$x^2=\frac{20}{3}=\frac{4\cdot 5}{3}$$

$$x=\pm 2\sqrt{\frac{5}{3}}$$

$P_1\left(2\sqrt{\frac{5}{3}}, \sqrt{\frac{7}{3}}\right)$

$P_2\left(2\sqrt{\frac{5}{3}}, -\sqrt{\frac{7}{3}}\right)$

$P_3\left(-2\sqrt{\frac{5}{3}}, \sqrt{\frac{7}{3}}\right)$

$P_4\left(-2\sqrt{\frac{5}{3}}, -\sqrt{\frac{7}{3}}\right)$

⑦ Dana je funkcija $f(x)=\frac{x+1}{x^2(x-1)}$. Zapišite njeno ničlo, pola in enačbo vodoravne asimptote. Narišite graf funkcije f v koordinatni sistem.

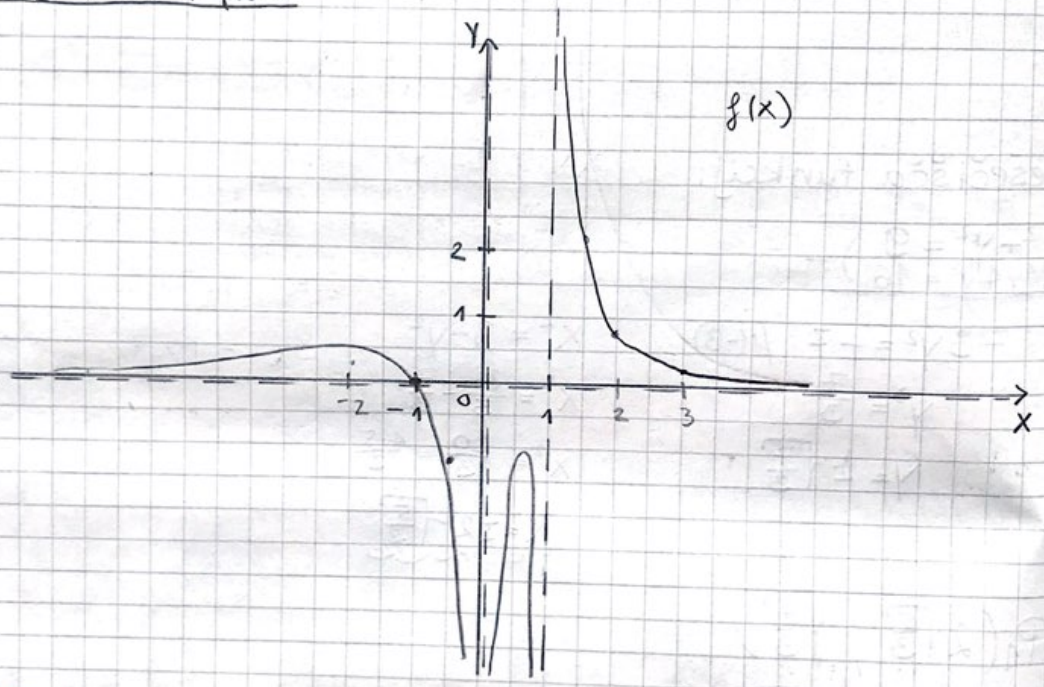
→ničla: $x+1=0$
$x=-1$ (1. st.)

→pola: $x^2(x-1)=0$
$x_1=0$ (2. st.)
$x_2=1$ (1. st.)

→vodoravna asimptota:

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{x+1\ /:x^3}{x^3-x^2\ /:x^3}=\lim_{x\to\infty}\frac{\frac{1}{x^2}+\frac{1}{x^3}}{1-\frac{1}{x}}=\frac{0}{1}=0$$

$y=0$ vod asimptota



✓ ⑨ Dana sta vektorja $\vec{a}=(2,-1,3)$ in $\vec{b}=(1,-2,5)$ v običajni bazi $\vec{i},\vec{j},\vec{k}$. Zapišite komponente vektorjev $\vec{x}=2\vec{a}-\vec{b}$ in $\vec{y}=\vec{a}+\vec{b}$. Izračunajte točno dolžino vektorja $\vec{x}$ in skalarni produkt vektorjev $\vec{x}\cdot\vec{y}$.

$\vec{a}=(2,-1,3)$, $\vec{b}=(1,-2,5)$

$\vec{x}=2(2,-1,3)-(1,-2,5)$
$\vec{x}=(4,-2,6)-(1,-2,5)$
$\vec{x}=(3,0,1)$ ✓

$\vec{y}=(2,-1,3)+(1,-2,5)$
$\vec{y}=(3,-3,8)$ ✓

! $|\vec{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2} = \sqrt{9+0+1} = \sqrt{10}$ ✓

$\vec{x} \cdot \vec{y} = x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 y_3$

$\vec{x} \cdot \vec{y} = 3 \cdot 3 + 0 \cdot (-3) + 1 \cdot 8 = 9 + 8 = 17$ ✓

✓ (10.) Za katera realna števila x so $2x, 1-x, x^2$ prvi trije členi GZ?

GZ: $2x, 1-x, x^2$

Kvocient med sosednjimi členi je vedno enak (GZ).

$a_1 = 2x = a_1$

$\frac{a_2}{a_1} = \frac{a_3}{a_2}$

$a_2 = 1-x = a_1 k$

$\frac{1-x}{2x} = \frac{x^2}{1-x} \quad / \cdot (1-x)2x$

$a_3 = x^2 = a_1 k^2$

$(1-x)^2 = 2x^3$

$1 - 2x + x^2 = 2x^3$

$2x^3 - x^2 + 2x - 1 = 0$

$2x(x^2+1) - (x^2+1) = 0$

$(x^2+1)(2x-1) = 0$

↓ ni $\mathbb{R}$ reš. ↓ $2x - 1 = 0$

$x = 1/2$ ✓

GZ: $1, 1/2, 1/4$ ✓

(11.) V kateri točki je tangenta na krivulji z enačbo $y = -x^3 - 3x^2 - 2x + 1$ vzporedna premici $y = x + 3$?

$t \parallel p \Leftrightarrow k_t = k_p$

$p: y = x + 3 \Rightarrow k_p = 1 \Rightarrow k_t = 1$

$k_t = y'(x_1)$

$y' = -3x^2 - 6x - 2 = 1$

$-3x^2 - 6x - 3 = 0 \quad / : (-3)$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x_1 = -1, \quad y_1 = -(-1)^3 - 3(-1)^2 - 2(-1) + 1 = 1$

$T(-1, 1)$

V točki T (-1, 1) je tangenta na krivuljo vzporedna premici $y = x + 3$.

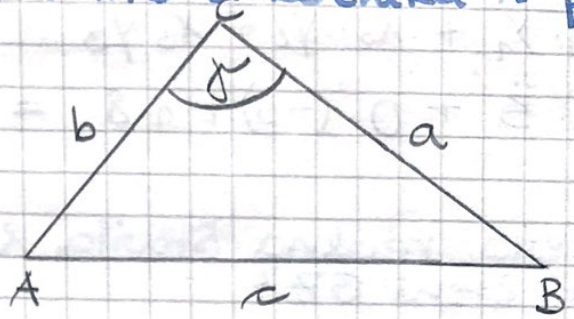
(12) Izračunaj stranici a in b trikotnika s podatki:

$S = 60\sqrt{3}\ cm^2$

$a : b = 3 : 5$

$\gamma = 60°$

$a, b = ?$



$S_\Delta = \frac{1}{2} a \cdot b \cdot \sin\gamma$

$a = (3x)\ cm$

$b = (5x)\ cm$

$60\sqrt{3}\ cm^2 = \frac{1}{2} 3x \cdot 5x\ cm^2 \cdot \sin 60°$

$60\sqrt{3} = \frac{1}{2} \cdot 15x^2 \cdot \frac{\sqrt{3}}{2} \quad / \cdot 4$

$240\sqrt{3} = 15x^2 \cdot \sqrt{3} \quad / : \sqrt{3}$

$240 = 15x^2$

$x^2 = \frac{240}{15} = 16 \Rightarrow x = \pm\sqrt{16} = 4$