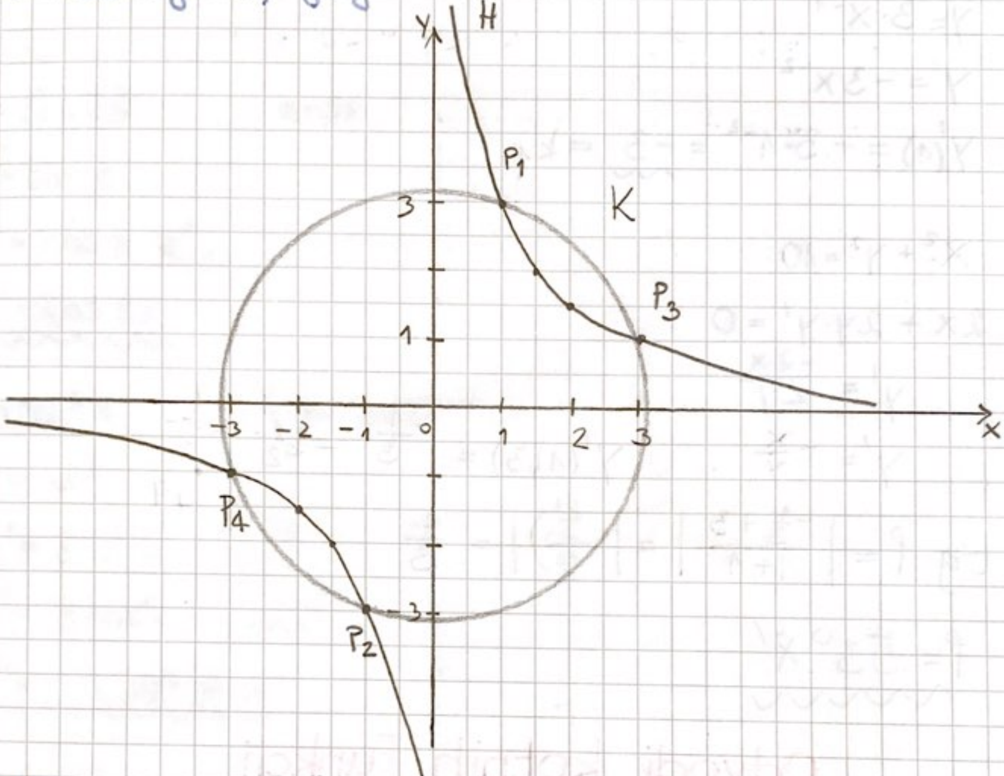
② $K: x^2+y^2=10$
$H: xy=3$ zasuk za 45° $y=\frac{3}{x}$ 

$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$

kot med njima, graf



Presečišče:

$x^2+y^2=10$
$xy=3 \Rightarrow x=\frac{3}{y}$

$\left(\frac{3}{y}\right)^2+y^2=10$

$\frac{9}{y^2}+y^2=10 \quad / \cdot y^2$

$9+y^4=10y^2$

$y^4-10y^2+9=0$

$(y^2-9)(y^2-1)=0$

$(y-3)(y+3)(y-1)(y+1)=0$

$y_1=3,\ y_2=-3,\ y_3=1,\ y_4=-1$

$x_1=\frac{3}{3}=1,\ x_2=\frac{3}{-3}=-1,\ x_3=\frac{3}{1}=3,\ x_4=\frac{3}{-1}=-3$

$P_1(1,3),\ P_2(-1,-3),\ P_3(3,1),\ P_4(-3,-1)$

→ kot med krivuljama v presečišču: $P_1(1,3)$

$$\operatorname{tg}\varphi = \left|\frac{k_2 - k_1}{1 + k_1 \cdot k_2}\right|$$

$$y = 3 \cdot x^{-1}$$

$$y' = -3x^{-2}$$

$$y'(1) = -3 \cdot 1^{-2} = -3 = k_1$$

$$x^2 + y^2 = 10$$

$$2x + 2y \cdot y' = 0$$

$$y' = \frac{-2x}{2y}$$

$$y' = -\frac{x}{y} \qquad y'(1,3) = -\frac{1}{3} = k_2$$

$$\operatorname{tg}\varphi = \left|\frac{-\frac{1}{3} + 3}{1 + 1}\right| = \left|\frac{\frac{\cancel{8}^{4}}{3}}{\frac{\cancel{2}}{1}}\right| = \frac{4}{3}$$

$$\varphi = 53^\circ\, 8'$$

17. V kateri točki krivulje $y=x^2+4x$ je tangenta vzporedna osi x?

os x: $y=0 \Rightarrow k=0$

$t \parallel$ os x $\Leftrightarrow kt = k = 0$

$kt = y'(x_1)$

$y' = 2x+4$

$2x+4=0$
$2x=-4$
$x=-2$ $\quad P(-2,-4)$

tangenta: $y=-2$

18. Zapiši enačbi tangent parabole $y=4-x^2$ v njenih presečiščih z osjo x.

Presečišča:

$y=4-x^2$

x os: $y=0 \rightarrow y=4-x^2$ $\quad y'=-2x$

$4-x^2=0$
$(2-x)(2+x)=0$
$x_1=2, \; x_2=-2$ $\quad P_1(2,0)$
$P_2(-2,0)$

t_1: $y-y_1=kt_1(x-x_1)$ $\quad P_1(2,0)$

$kt_1=y'(2)$ $\quad kt_1=-4$

$y-0=-4(x-2)$
$y=-4x+8$

t_2: $y-y_2=kt_2(x-x_2)$ $\quad P_2(-2,0)$

$y'(-2)=4=kt_2$

$y-0=4(x+2)$
$y=4x+8$

(20.) Ali ima krivulja $y = \frac{x}{x-2}$ tangento z naklonskim kotom 45°?

$\varphi = 45°$ $\quad tg\,\varphi = k_t$ $\quad tg\,45° = 1 = k_t$

$y' = \frac{(x)'(x-2) - x(x-2)'}{(x-2)^2} = \frac{\cancel{x}-2-\cancel{x}}{(x-2)^2}$

$y' \overset{?}{=} k_t$

$\frac{-2}{(x-2)^2} \overset{?}{=} 1 \quad /\cdot (x-2)^2$

$-2 = (x-2)^2$

$-2 = x^2 - 4x + 4$

$x^2 - 4x + 6 = 0$

$D = (-4)^2 - 4 \cdot 1 \cdot 6 = 16 - 24 = -8$

$D < 0 \Rightarrow$ ni realni rešitev

→ Krivulja nima tangente s tem naklonskim kotom.

→ Določi enačbo tangente na krivuljo, graf:

$3x^2 + 7y^2 = 55$, v točki $T(x>0, 2)$.

tangenta: $y - y_1 = k_t(x - x_1)$

$k_t = y'(x_1)$

odvod: $6x + 14y' \cdot y = 0$

$y' = -\frac{6x}{14y} = -\frac{3x}{7y}$

T: $3x^2 + 7 \cdot 2^2 = 55$

$3x^2 + 28 = 55$

$3x^2 = 27 \quad /:3$

$x^2 = 9$

$x_1 = 3, \; x_2 = -3$

$x > 0$

$55 - 28 = 27$

$k_t = -\frac{3 \cdot 3}{7 \cdot 2} = -\frac{9}{14}$

t: $y - 2 = -\frac{9}{14}(x - 3)$

$y = -\frac{9}{14}x + \frac{27}{14} + \frac{28}{14}$

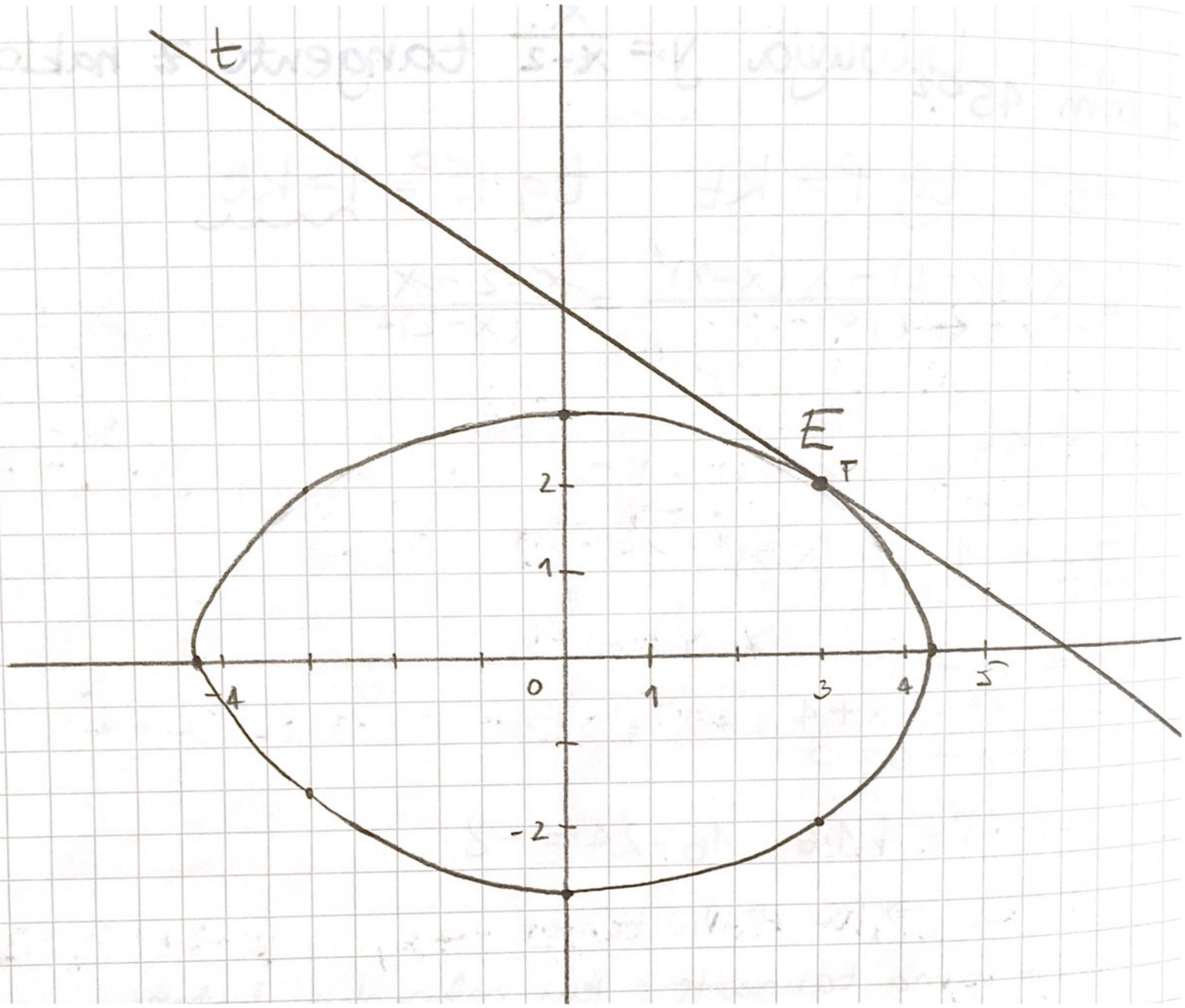
$y = -\frac{9}{14}x + \frac{55}{14}$ $\qquad 9x + 14y - 55 = 0$

ELIPSA:

$3x^2 + 7y^2 = 55 \quad /:55$

$\frac{3x^2}{55} + \frac{7y^2}{55} = 1$

$a = \sqrt{\frac{55}{3}} \doteq 4{,}3, \quad b = \sqrt{\frac{55}{7}} \doteq 2{,}8$


t
E
T
2
1
-2
-4
0
1
3
4
5

① $x^2-y^2-2x-4y-3=0$ HIPERBOLA
$x^2+y^2-4x+4y+3=0$ KROŽNICA

φ - kot med krivuljama v presečišču, ki ne leži na koordinatnih oseh

→ PRESEČIŠČE:

$$\left.\begin{array}{l} x^2-y^2-2x-\cancel{4y}-3=0 \\ x^2+y^2-4x+\cancel{4y}+3=0 \end{array}\right)+$$

0, -3
0, -1
3, 0
$P(3,-4)$

$2x^2-6x=0$
$2x(x-3)=0$
$x_1=0,\ x_2=3$

$x=3$
$y^2+4y=x^2-2x-3$
$y^2+4y=3^2-2\cdot 3-3$
$y^2+4y=9-6-3$
$y^2+4y=0$
$y(y+4)=0$
$y_1=0,\ y_2=-4 \Rightarrow P_1(3,-4)$

$$tg\,\varphi=\left|\frac{k_2-k_1}{1+k_1\cdot k_2}\right|$$

t_1: $y-y_1=kt_1(x-x_1)$ $P_1(3,-4)$, $x^2-y^2-2x-4y-3=0$

$kt_1=y'(P)$

odvod: $2x-2y\cdot y'-2-4y'=0$
$2x-2=2y\cdot y'+4y'$ /:2
$x-1=y\cdot y'+2y'$
$y'(y+2)=x-1$
$y'=\frac{x-1}{y+2}$ $kt_1=\frac{3-1}{-4+2}=\frac{2}{-2}=-1$

$kt_2=y'(P)$ $x^2+y^2-4x+4y+3=0$

odvod: $2x+2y\cdot y'-4+4y'=0$ /:2
$x+y\cdot y'-2+2y'=0$
$y'(y+2)=2-x$
$y'=\frac{2-x}{y+2}=\frac{2-3}{-4+2}=\frac{-1}{-2}=\frac{1}{2}$

$kt_2=\frac{1}{2}$

$$tg\,\varphi=\left|\frac{\frac{1}{2}+1}{1-\frac{1}{2}}\right|=\left|\frac{\frac{3}{2}}{\frac{1}{2}}\right|=3$$

$\varphi=71°34'$

t_1: $y=-x-3+4$
$y=-x-1$

t_2: $y=\frac{1}{2}x-\frac{3}{2}-4$
$y=\frac{1}{2}x-\frac{11}{2}$

HIPERBOLA: (?)

$$x^2-y^2-2x-4y-3=0$$
$$x^2-2x-y^2-4y-3=0$$
$$(x-1)^2-1-(y^2+4y)-3=0$$
$$(x-1)^2-(y+2)^2+4-1-3=0$$
$$(x-1)^2-(y+2)^2=0 \quad \text{razstavimo}$$
$$(x-1-y-2)(x-1+y+2)=0$$

enačbi dveh premic

HIP.

$$(\quad)^2-(\quad)^2=0$$

↓ sekajoči 2 premici (unija dveh premic)

PONOVI POSEBNI PRIMERI KRIVULJ 2. REDA

p_1: $x-y-3=0$

$y=x-3$

p_2: $x+y+1=0$

$y=-x-1$

$R: p_1 \cup p_2$

UNIJA

PREMAKNJENA KROŽNICA:

$$x^2+y^2-4x+4y+3=0$$
$$(x-2)^2-4+(y+2)^2-4+3=0$$
$$(x-2)^2+(y+2)^2=5$$

$S(2,-2)$, $r=\sqrt{5}$