Vaja

list

12.c) $\int \frac{x^2 \cdot \sqrt[4]{x}}{\sqrt[5]{x^7}}\,dx = \int x^{2+\frac{1}{4}-\frac{7}{5}}\,dx = \int x^{\frac{17}{20}}\,dx = \frac{20}{37} x^{\frac{37}{20}} + C$

$= \frac{20}{37} \sqrt[20]{x^{37}} + C$

e) $\int (3\sqrt{x} - x^{-5} + 2)\,dx = 3\int x^{\frac{1}{2}}\,dx - \int x^{-5}\,dx + 2\int dx =$

$= \cancel{3} \cdot \frac{2}{\cancel{3}} x^{\frac{3}{2}} + \frac{1}{4} x^{-4} + 2x + C = 2x\sqrt{x} + \frac{1}{4x^4} + 2x + C$

b) $\int \sqrt[3]{x}\,dx = \int x^{\frac{1}{3}}\,dx = \frac{3}{4} x^{\frac{4}{3}} + C = \frac{3}{4} \sqrt[3]{x^4} + C$

d) $\int (2x - 3x^4)\,dx = 2\int x\,dx - 3\int x^4\,dx = 2 \cdot \frac{1}{2} x^2 - 3 \frac{1}{5} x^5 + C =$

$= x^2 - \frac{3}{5} x^5 + C$

a) $\int x^3\,dx = \frac{1}{4} x^4 + C$

j) $\int (2\sin x + 5\cos x)\,dx = 2\int \sin x\,dx + 5\int \cos x\,dx = 2 \cdot (-\cos x) +$

$5 \cdot \sin x + C = -2\cos x + 5\sin x + C$

k) $\int \frac{\sin 2x}{\sin x \cos^3 x}\,dx = \int \frac{2 \cdot \cancel{\sin x} \cdot \cancel{\cos x}}{\cancel{\sin x} \cdot \cancel{\cos^3 x}}\,dx = \int \frac{2}{\cos^2 x}\,dx =$

$= 2\int \frac{1}{\cos^2 x}\,dx = 2\,\mathrm{tg}\,x + C$

l) $\int \frac{\cos 2x}{\sin^2 x}\,dx = \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x}\,dx = \int \frac{1 - \sin^2 x - \sin^2 x}{\sin^2 x}\,dx =$

$= \int \frac{1 - 2\sin^2 x}{\sin^2 x}\,dx = \int \frac{1}{\sin^2 x} - 2\int dx = -\mathrm{ctg}\,x - 2x + C$

f) $\int (2x-1)\sqrt[3]{x}\,dx = \int 2x\sqrt[3]{x}\,dx - \int \sqrt[3]{x}\,dx =$

$= 2\int x^{\frac{4}{3}}\,dx - \int x^{\frac{1}{3}}\,dx = 2\frac{3}{7} x^{\frac{7}{3}} - \frac{3}{4} x^{\frac{4}{3}} + C =$

$= \frac{6}{7} x^2 \sqrt[3]{x} - \frac{3}{4} x \sqrt[3]{x} + C$

m) $\int \frac{2^x+3^x}{5^x}dx = \int \frac{2^x}{5^x}dx + \int \frac{3^x}{5^x}dx = \int \left(\frac{2}{5}\right)^x dx + \int \left(\frac{3}{5}\right)^x dx =$

$= \frac{\left(\frac{2}{5}\right)^x}{\ln\frac{2}{5}} + \frac{\left(\frac{3}{5}\right)^x}{\ln\frac{3}{5}} + C$

$\int (\cos 3x + \sin 5x - e^{2x})\,dx = \int \cos 3x \cdot dx + \int \sin 5x - \int e^{2x} =$

$= \frac{\sin 3x}{3} - \frac{\cos 5x}{5} \; \frac{e^{2x}}{2} + C$

$\int (\sin x \cdot \cos x)\,dx = \int \frac{\sin 2x}{2}dx = \frac{1}{2}\int \sin 2x\,dx =$

$= \frac{1}{2}\left(\frac{-\cos 2x}{2}\right) + C = \frac{-\cos 2x}{4} + C$

i) $\int (3e^x + 5)dx = 3\int e^x dx + 5\int dx = 3e^x + 5x + C$

k) $\int \frac{3x - 2\cdot\sqrt[3]{x}}{\sqrt[5]{x}}dx = 3\int x^{1-\frac{1}{5}}dx - 2\int x^{\frac{1}{3}-\frac{1}{5}}dx = 3\int x^{\frac{4}{5}}dx - 2\int x^{\frac{2}{15}}dx =$

$= \cancel{3}\frac{5}{\cancel{9}_3}x^{\frac{9}{5}} - 2\cdot\frac{15}{17}x^{\frac{17}{15}} + C = \frac{5}{3}x\sqrt[5]{x^4} - \frac{30}{17}x\sqrt[15]{x^2} + C$

g) $\int \frac{2x^3+3x-2}{x^2}dx = 2\int x\,dx + 3\int x^{-1}dx - 2x^{-2}dx =$

$= \cancel{2}\cdot\frac{1}{\cancel{2}}x^2 + 3\ln(x) - 2(-1)x^{-1} + C =$

$= x^2 + 3\ln|x| + \frac{2}{x} + C$

formula priložena na maturi

$\int \sin x \cdot \cos 2x\,dx =$ RAZČLENITEV PRODUKTA !!!

$= \int \frac{1}{2}[\sin(x+2x) + \sin(x-2x)]dx = \frac{1}{2}\int(\sin 3x + \sin(-x))dx =$

$= \frac{1}{2}\int \sin 3x - \frac{1}{2}\int \sin x = \frac{1(-\cos 3x)}{2\cdot 3} - \frac{1}{2}(-\cos x) + C =$

$= -\frac{1}{6}\cos 3x + \frac{1}{2}\cos x + C$

Integriranje z uvedbo nove spremenljivke

$\int \sin 5x \, dx = \int \sin z \, \frac{1}{5} dz$

$z = 5x \quad , z' = 5$

$dz = z' \cdot dx$

$dz = 5 \cdot dx$

$= \frac{1}{5} \int \sin z \, dz = \frac{1}{5} (-\cos z) + C$

$= -\frac{1}{5} \cos 5x + C$

$3x = z \ , \ z' = 3 \ , \ dz = z' dx \ , \ dz = 3\,dx \ , \ dx = \frac{dz}{3}$

$\int (3x)^{-1} dx = \int z^{-1} \frac{1}{3} dz = \frac{1}{3} \int z^{-1} dz = \frac{1}{3} \ln|z| + C =$

$= \frac{1}{3} \ln|3x| + C$

$\int \sqrt[3]{5x-1} \, dx = \int z^{\frac{1}{3}} \frac{1}{5} dz = \frac{1}{5} \int z^{\frac{1}{3}} dz = \frac{1}{5} \cdot \frac{3}{4} z^{\frac{4}{3}} + C =$

$z = 5x - 1$

$z' = 5$

$dz = z' \cdot dx$

$dz = 5 \cdot dx$

$dx = \frac{1}{5} dz$

$= \frac{3}{20} z \sqrt[3]{z} + C = \frac{3}{20} (5x-1) \sqrt[3]{5x-1} + C$

$= \left(\frac{15}{20} x - \frac{3}{20}\right) \sqrt[3]{5x-1} + C$

$\int \sin^3 x \, dx = \int z^3 \frac{dz}{\cos x}$

ne tako!

$\sin x = z$

$z' = \cos x$

$dz = z' \cdot dx$

$dz =$

$\int \cos^2 x \cdot \sin x \, dx = \int t^2 \cdot \cancel{\sin x} \frac{-dt}{\cancel{\sin x}} = -\int t^2 dt =$

$t = \cos x$

$dt = -\sin x \cdot dx$

$dx = \frac{-dt}{\sin x}$

$= -\frac{1}{3} t^3 + C = -\frac{1}{3} \cos^3 x + C$

LEPŠI ZAPIS:

$\int \cos^2 x \cdot \sin x \, dx = \int t^2 (-dt) = -\int t^2 dt =$

$t = \cos x$

$dt = -\sin x \cdot dx$

$\sin x \cdot dx = -dt$

$\int x e^{x^2} dx = \int e^z \cdot \frac{dz}{2} = \frac{1}{2} \int e^z dz = \frac{1}{2} e^z + C =$

$x^2 = z$

$dz = 2x \cdot dx$

$x \cdot dx = \frac{dz}{2}$

$= \frac{1}{2} e^{x^2} + C$

1. $\int \sin x \cdot \sqrt{\cos x}\, dx = \int \cancel{\sin x} \cdot t^{\frac{1}{2}} \cdot -\frac{dt}{\cancel{\sin x}} = -\frac{2}{3} t^{\frac{3}{2}} + C =$

$= -\frac{2}{3} t \sqrt{t} + C = -\frac{2}{3} \cos x \sqrt{\cos x} + C$

$\cos x = t$
$t' = -\sin x$
$dt = -\sin x \cdot dx$
$dx = -\frac{dt}{\sin x}$

2. $\int x \cdot e^{x^2} dx = \int \cancel{x} e^t \cdot \frac{dt}{2\cancel{x}} = \frac{1}{2} \int e^t dt = \frac{1}{2} \cdot e^t + C =$

$= \frac{1}{2} e^{x^2} + C$

$x^2 = t$
$t' = 2x$
$dt = 2x \cdot dx$
$dx = \frac{dt}{2x}$

3. $\int \frac{\ln x}{x} dx = \int z \cdot dz = \frac{1}{2} z^2 + C = \frac{1}{2} \ln^2 x + C$

$\ln x = z$
$z' = \frac{1}{x}$
$dz = \frac{1}{x} \cdot dx$

4. $\int \cos(ax-b)\, dx = \int \cos u \cdot \frac{du}{a} = \frac{1}{a} \int \cos u \cdot du = \frac{1}{a} \sin u + C =$

$ax - b = u$ $\quad = \frac{1}{a} \sin(ax-b) + C$
$du = a \cdot dx$
$dx = \frac{du}{a}$

5. $\int e^{\sin x} \cdot \cos x\, dx = \int e^z \cdot dz = e^z + C = e^{\sin x} + C$

$\sin x = z$
$z' = \cos x$
$dz = \cos x \cdot dx$

6. $\int \frac{\cos x}{1 + \sin x} dx = \int \frac{1}{t} \cdot dt = \ln|t| + C = \ln|\sin x + 1| + C$

$\sin x + 1 = t$
$t' = \cos x$
$dt = \cos x \cdot dx$

$$\boxed{\int u \cdot dv = u \cdot v - \int v\, du}$$

Zgledi:

① $\int x \cdot e^x dx = x \cdot e^x - \int e^x dx = \underbrace{x \cdot e^x - e^x + C}$

algeb. trans.

odvajam

$u = x \rightarrow du = u' \cdot dx = 1 \cdot dx = dx$

$e^x dx = dv \rightarrow v = \int e^x dx = e^x + C$

integriram

NAPAČNO!

② $\int x \cdot e^x dx = e^x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot e^x \cdot dx = \dots$

$u = e^x \rightarrow du = e^x \cdot dx$

$dv = x \cdot dx \rightarrow v = \int x \cdot dx = \frac{1}{2}x^2$

③ $\int \ln x\, dx = \ln x \cdot x - \int x \cdot \frac{1}{x} dx = x \cdot \ln x - x + $

$u = \ln x \quad , \quad du = \frac{1}{x} dx$

$dv = dx \quad , \quad v = \int dx = x + C$

④ $\int x^2 \cdot \sin x\, dx = x^2 \cdot (-\cos x) - \int (-\cos x) \cdot 2x \cdot dx =$

$u = x^2 \quad , \quad du = 2x \cdot dx$

$dv = \sin x\, dx \quad , \quad v = \int \sin x\, dx = -\cos x + C$

$= -x^2 \cdot \cos x + 2 \int x \cdot \cos x\, dx =$

še enkrat per partes

$u = x \Rightarrow du = dx$

$dv = \cos x\, dx \Rightarrow v = \int \cos x\, dx = \sin x + C$

$= -x^2 \cdot \cos x + 2\left(x \cdot \sin x - \int \sin x \cdot dx\right) =$

$= -x^2 \cdot \cos x + 2x \cdot \sin x - 2\cos x + C$

5. $\int x \cdot \ln x\, dx = \ln x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx = \frac{1}{2}x^2 \ln x - \int \frac{1}{2}x\, dx =$

$u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = x \cdot dx \Rightarrow v = \int x \cdot dx = \frac{1}{2}x^2 + C$

$= \frac{1}{2}x^2 \cdot \ln x - \frac{1}{2} \cdot \frac{1}{2}x^2 + C =$

$= \frac{1}{2}x^2 \cdot \ln x - \frac{1}{4}x^2 + C$

$= \frac{1}{4}x^2 (2\ln x - 1) + C$