11. $\int (1-\sin^2 x)^{-1}\, dx = \int \frac{1}{1-\sin^2 x}\, dx = \int \frac{1}{\cos^2 x}\, dx = tg\, x + C$

12. $\int (1+ctg^2 x)\, dx = \int \left(\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}\right) dx = \int \frac{1}{\sin^2 x} = -ctg\, x + C$

13. $\int 2^x\, dx = 2^x \frac{1}{\ln 2} + C$

14. $\int (1-t^2)^{-\frac{1}{2}}\, dt = \int \frac{1}{\sqrt{1-t^2}}\, dt = arc\sin t + C$

15. $\int (1+t^2)^{-1}\, dt = \int \frac{1}{1+t^2}\, dt = arc\, tg\, t + C$

16. $\int ctg\, t \cdot dt = \ln(\sin t) + C$

17. $\int \frac{\sin 2x}{2 \sin x}\, dx = \int \frac{\cancel{2 \sin x} \cos x}{\cancel{2 \sin x}}\, dx = \sin x + C$

19. $\int \frac{\sqrt{x}}{\sqrt[3]{x}}\, dx = \int x^{\frac{1}{2}-\frac{1}{3}}\, dx = \int x^{\frac{1}{6}}\, dx = \frac{6}{7} x^{\frac{7}{6}} + C = \frac{6}{7} x \sqrt[6]{x} + C$

20. $\int \frac{x \sqrt[5]{x^3}}{\sqrt{x^3}}\, dx = \int x^{1+\frac{3}{5}-\frac{3}{2}}\, dx = \int x^{\frac{1}{10}}\, dx = \frac{10}{11} x^{\frac{11}{10}} + C = \frac{10}{11} x \sqrt[10]{x} + C$

22. $\int e^{\ln x}\, dx = \int x\, dx = \frac{1}{2} x^2 + C$, za $x > 0$

$\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2}$

(24) $\int \log_3 \sqrt[x]{3}\, dx = \int \log_3 3^{\frac{1}{x}} dx = \int \frac{1}{x}\, dx = \ln x + C,\ x \neq 0$

31.3.04 DOMAČA NALOGA

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(1) $\int \sin(3x)\cdot\cos(2x)\, dx = \int \frac{1}{2}(\sin 5x + \sin x)\, dx = \frac{1}{2}\int \sin 5x + \frac{1}{2}\int \sin x =$

$= \frac{1}{2}\cdot(-\cos 5x)\cdot\frac{1}{5} + \frac{1}{2}\cdot(-\cos x) + C = -\frac{1}{10}\cos 5x - \frac{1}{2}\cos x + C$

(2) $\int \sin(2x)\cdot\sin(3x)\, dx = \int \frac{1}{2}(\cos(-x) - \cos(5x))\, dx = \frac{1}{2}\int \cos x\, dx - \frac{1}{2}\int \cos 5x\, dx =$

$= \frac{1}{2}\sin x - \frac{1}{2}\cdot\frac{1}{5}\sin 5x + C = \frac{1}{2}\sin x - \frac{1}{10}\sin 5x + C$

(3) $\int \cos(2x-30°)\cdot\cos(x+60°)\, dx = \int \frac{1}{2}(\cos(3x+30°) + \cos(x-90°))\, dx =$

$= \frac{1}{2}\left(\frac{1}{3}\sin(3x+30°) + \sin(x-90°)\right) + C = \frac{1}{6}\sin(3x+30°) - \frac{1}{2}\cos x + C$

(1) $\int \sin(2x)\cdot\cos(4x)\, dx = \frac{1}{2}\int(\sin 6x + \sin(-2x))\, dx = \frac{1}{2}\int \sin 6x\, dx - \frac{1}{2}\int \sin 2x\, dx =$

$= \frac{1}{2}\cdot\frac{1}{6}\cdot(-\cos 6x) - \frac{1}{2}\cdot\frac{1}{2}(-\cos 2x) + C = -\frac{1}{12}\cos 6x + \frac{1}{4}\cos 2x + C$

(3) $\int \sin(5x)\cdot\sin x\, dx = \frac{1}{2}\int(\cos(4x) - \cos(6x))\, dx = \frac{1}{2}\int \cos(4x)\, dx -$

$\frac{1}{2}\int \cos(6x)\, dx = \frac{1}{8}\sin(4x) - \frac{1}{12}\sin(6x) + C$

(4) $\int \cos(2x)\cdot\cos(4x)\, dx = \frac{1}{2}\int(\cos(6x) + \cos(-2x))\, dx = \frac{1}{2}\int \cos(6x)\, dx +$

$\frac{1}{2}\int \cos(2x)\, dx = \frac{1}{12}\sin(6x) + \frac{1}{4}\sin(2x) + C$

(5) $\int \cos x\cdot\cos(3x)\, dx = \frac{1}{2}\int(\cos(4x) + \cos(-2x))\, dx = \frac{1}{2}\int \cos(4x)\, dx +$

$\frac{1}{2}\int \cos(2x)\, dx = \frac{1}{8}\sin 4x + \frac{1}{4}\sin(2x) + C$

(7) $\int \sin^2(2x)\, dx = \frac{1}{2}\int(1-\cos 2z)\,\frac{dz}{2} = \frac{1}{4}\int(1-\cos 2z)\, dz =$

$z = 2x$
$z' = 2\ ,\ dz = z'\cdot dx \Rightarrow dx = \frac{dz}{2}$

$$= \frac{1}{4}\int dz - \frac{1}{4}\int \cos 2z \, dz = \frac{1}{4} 2x - \frac{1}{8}\sin 2z + C = \frac{1}{2}x - \frac{1}{8}\sin 4x + C$$

8. $$\int \cos^2(3x)\,dx = \frac{1}{2}\int (1+\cos 2z)\frac{dz}{3} = \frac{1}{6}\int dz + \frac{1}{6}\int \cos 2z\, dz =$$

$z = 3x$, $z' = 3$
$dz = z'dx = 3\cdot dx$
$dx = \frac{dz}{3}$

$$= \frac{1}{6}z + \frac{1}{12}\sin 2z + C = \frac{1}{\cancel{6}_2} \cancel{3}x + \frac{1}{12}\sin 6x + C =$$

$$= \frac{1}{2}x + \frac{1}{12}\sin 6x + C$$

9. $$\int \sin(2x)\cdot\cos^{-4}x\,dx = \int 2\cdot\sin x\cdot\cancel{\cos x}\cdot\cos^{-4}x\,dx = 2\int \sin x\cdot\cos^{-3}x\,dx =$$

$$= 2\int \frac{\sin x}{\cos x}\cdot\frac{1}{\cos^2 x}\,dx = 2\int tg x\cdot\frac{1}{\cos^2 x}\,dx =$$

$t = tg x$
$t' = \frac{1}{\cos^2 x}$
$dt = \frac{1}{\cos^2 x}dx$

$$= 2\int t\cdot dt = 2\cdot\frac{1}{2}t^2 + C = tg^2 x + C$$

10. $$\int \cos^3 x\cdot tg x\,dx = \int \frac{\cos^{\cancel{3}\,2} x}{1}\cdot\frac{\sin x}{\cancel{\cos x}}\,dx = \int \cos^2 x\cdot\sin x\,dx =$$

$t = \cos x$, $t' = -\sin x$
$dt = -\sin x\cdot dx$

$$= -\int t^2\cdot dt = -\frac{1}{3}t^3 + C = -\frac{1}{3}\cos^3 x + C$$

③ $$\int\left(\frac{\sin^2 x}{\cos^2 x}\right)dx = \int\left(\frac{1-\cos^2 x}{\cos^2 x}\right)dx = tg x - x + c$$

11. $$\int tg^2 x\,dx \overset{①}{=} \int\left(\frac{1}{\cos^2 x} - 1\right)dx = \int\frac{1}{\cos^2 x}dx - \int dx = tg x - x + c$$

① $\int (\cos x + 2e^{2x} + 3^x)\,dx = \int \cos x\,dx + 2\int e^{2x}\,dx + \int 3^x\,dx =$

$= \sin x + 2\cdot e^{2x}\cdot\frac{1}{2} + 3^x\cdot\frac{1}{\ln 3} + C = \sin x + e^{2x} + \frac{3^x}{\ln 3} + C$

② $\int \left(\sqrt[4]{x^3} - \frac{2}{\sqrt{x}} + \frac{1}{x}\right) dx = \int \quad x - 2\int x^{-\frac{1}{2}}\,dx + \int x^{-1}\,dx =$

$= \frac{4}{7}x^{\frac{7}{4}} - 2\cdot\frac{2}{1}\cdot x^{\frac{1}{2}} + \ln|x| + C = \frac{4}{7}x\sqrt[4]{x^3} - 4\sqrt{x} + \ln|x| + C$

③ $\int \frac{1-x^3-x^{\frac{1}{3}}}{x^2}\,dx = \int x^{-3}\,dx - \int dx - \int x^{-\frac{8}{3}}\,dx = -\frac{1}{2}x^{-2} - x + \frac{3}{5}x^{-\frac{5}{3}} +$

$C = -\frac{1}{2x^2} - x + \frac{3}{5}\cdot\frac{1}{x\sqrt[3]{x^2}} + C$

④ $\int \frac{1-\sin^3 x}{\sin^2 x}\,dx = \int \left(\frac{1}{\sin^2 x} - \frac{\sin^3 x}{\sin^2 x}\right) dx = \int \frac{1}{\sin^2 x}\,dx - \int \sin x\,dx =$

$= -\operatorname{ctg} x + \cos x + C$

⑤ $\int \cos^3 x\,dx = \int \cos^2 x\cdot\cos x\,dx = \int ((1-\sin^2 x)\cdot\cos x)\,dx =$

$= \int \cos x\,dx - \int \sin^2 x\cdot\cos x\,dx = \sin x + C - \int t^2\cdot dt =$

$\sin x = t$
$t' = \cos x$
$dt = \cos x\cdot dx$

$= \sin x + C - \frac{1}{3}t^3 = \sin x + C - \frac{1}{3}\sin^3 x$

$= \sin x - \frac{1}{3}\sin^3 x + C$

$\int x^2\cdot\ln x\,dx = \ln x\cdot\frac{1}{3}x^3 - \int \frac{1}{3}x^{\cancel{3}\,2}\cdot\frac{1}{x}\,dx = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx$

$u = \ln x \Rightarrow du = u'\cdot dx = \frac{1}{x}\cdot dx$
$dv = x^2\cdot dx \Rightarrow v = \int x^2\,dx = \frac{1}{3}x^3 + C$

$= \frac{x^3}{3}\ln x - \frac{1}{9}x^3 + C$

$= \frac{x^3}{9}(3\ln x - 1) + C$