1\. a) $\lim\limits_{x\to\infty} \dfrac{(2x-3)^3}{4x-x^3} = \lim\limits_{x\to\infty} \dfrac{8x^3+3\cdot 4x^2\cdot(-3)+3\cdot 2x\cdot 9-27}{-(x^3-4x)} =$

$= \lim\limits_{x\to\infty} \dfrac{8x^3-36x^2+54x-27 \;/:x^3}{-x^3+4x \;/:x^3} = \lim\limits_{x\to\infty} \dfrac{8-\frac{36}{x}+\frac{54}{x^2}-\frac{27}{x^3}}{-1+\frac{4}{x^2}} = \underline{\underline{-8}}$

b) $\lim\limits_{x\to -1} \dfrac{(\sqrt{x+2}-1)(\sqrt{x+2}+1)}{(x+1)(\sqrt{x+2}+1)} = \lim\limits_{x\to -1} \dfrac{x+2-1}{x\sqrt{x+2}+x+\sqrt{x+2}+1} =$

$= \lim\limits_{x\to -1} \dfrac{\cancel{(x+1)}\cdot 1}{\cancel{(x+1)}(\sqrt{x+2}+1)} = \lim\limits_{x\to -1} \dfrac{1}{\sqrt{x+2}+1} = \underline{\underline{\dfrac{1}{2}}}$

c) $\lim\limits_{x\to 0} \dfrac{1-\cos 2x}{x\cdot\sin x} = \lim\limits_{x\to 0} \dfrac{1-\cos^2 x+\sin^2 x}{x\cdot\sin x} = \lim\limits_{x\to 0} \dfrac{\sin^2 x+\sin^2 x}{x\cdot\sin x} =$

$= \lim\limits_{x\to 0} \dfrac{2\sin^2 x}{\underbrace{x\cdot\sin x}_{=1}} = \lim\limits_{x\to 0} \dfrac{2}{1}\cdot 1 = \underline{\underline{2}}$

2\. $f(x)=ax^2+bx+c$

$g(x)=x^{-1}$

a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{a\cdot(x+h)^2+b\cdot(x+h)+c-ax^2-b}{h}$

$= \lim\limits_{h\to 0} \dfrac{a(x^2+2xh+h^2)+bx+bh+c-ax^2-bx-c}{h} =$

$= \lim\limits_{h\to 0} \dfrac{\cancel{ax^2}+a2xh+ah^2+\cancel{bx}+bh+\cancel{c}-\cancel{ax^2}-\cancel{bx}-\cancel{c}}{h} =$

$= \lim\limits_{h\to 0} \dfrac{\cancel{h}\cdot(2ax+ah+b)}{\cancel{h}} = \lim\limits_{h\to 0}(2ax+ah+b) = \underline{\underline{2ax+b}}$

$f'(x)=0=2ax+b$

$2ax=-b$

$x=\dfrac{-b}{2a}$ → RAVNO ABSCISA TEMENA KVADRATNE FUNKCIJE

b) $f(g(x)) = f(x^{-1}) = a\cdot(x^{-1})^2 + bx^{-1} + c =$

$= ax^{-2} + bx^{-1} + c$

$g(f(1)) = g(a+b+c) = (a+b+c)^{-1} = \dfrac{1}{a+b+c}$

c) $g^{-1}(x) = ?$

$x \Leftrightarrow y$

$x = y^{-1}$

$x = \dfrac{1}{y} \quad / \cdot y$

$xy = 1$

$y = \dfrac{1}{x}$

$g^{-1}(x) = \dfrac{1}{x} = x^{-1}$

3. $f(x) = 2x^2 - 3x$

a) $T_1(-1, y_1)$ $\qquad y_1 = 2\cdot(-1)^2 - 3\cdot(-1) = 2 + 3 = 5$

$T_2(3, y_2)$ $\qquad y_2 = 2\cdot 3^2 - 3\cdot 3 = 9$

$T_1(-1, 5)$

$T_2(3, 9)$

Sekanta: $y - y_1 = k(x - x_1)$

$9 - 5 = k(3 + 1)$

$4 = 4k$

$k = 1$

$y = kx + n$

$9 = 1\cdot 3 + n$

$n = 6$

Enačba sekante: $y = x + 6$

b) $f(x) = 2x^2 - 3x$
$p: y = -2x + 3$

Presečišče: $y = y$

$$2x^2 - 3x = -2x + 3$$
$$2x^2 - x - 3 = 0$$
$$(2x - 3)(x + 1) = 0$$

$x_1 = -1$ $\quad P_1(-1, 5)$

$x_2 = \frac{3}{2}$ $\quad P_2(\frac{3}{2}, 0)$

$f'(x) = 4x - 3$

$f'(-1) = -4 - 3 = -7 = k_{t_1}$ ~~$= \tan\alpha$~~

$y' = -2 = k_2$

$$\tan\alpha = \left|\frac{k_2 - k_1}{1 + k_2 \cdot k_1}\right| = \left|\frac{-2+7}{1+14}\right| = \left|\frac{5}{15}\right| = \frac{5}{15}$$

$\tan\alpha = \frac{1}{3}$

$\alpha = 18{,}43°$

4. $f(x) = (-x^2 - 2) \cdot (5x^3 + 1)$

$f'(x) = (-x^2 - 2)' \cdot (5x^3 + 1) + (-x^2 - 2) \cdot (5x^3 + 1)' =$

$= (-2x) \cdot (5x^3 + 1) + (-x^2 - 2) \cdot (15x^2) =$

$= -10x^4 - 2x - 15x^4 - 30x^2 =$

$= -25x^4 - 30x^2 - 2x$

5. $p(x) = 2x^3 - 3x^2$

a) <u>Nicle</u> : $2x^3 - 3x^2 = 0$

$x^2 \cdot (2x - 3) = 0$

$x_{1,2} = 0 \quad x_3 = \frac{3}{2}$

<u>Ekstremi</u> : $f'(x) = 0$

$6x^2 - 6x = 0$

$x^2 - x = 0$

$x \cdot (x-1) = 0$

$x_1 = 0 \quad x_2 = 1$

$y_1 = 0 \quad y_2 = -1$

$T_{max}(0,0) \longrightarrow$

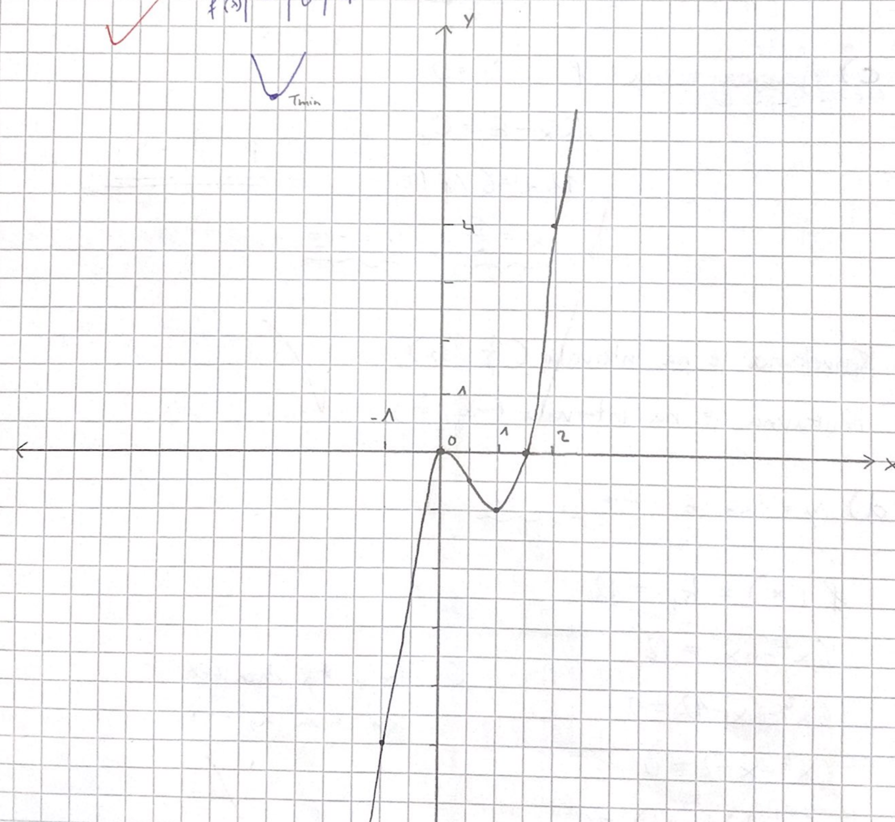
x	-1	0	$\frac{1}{2}$
$f'(x)$	$+$	0	$-$

T_{max}

$T_{min}(1,-1) \longrightarrow$

x	$\frac{1}{2}$	1	2
$f'(x)$	$-$	0	$+$

T_{min}



b) $T(-2, y_0)$

$y_0 = 2\cdot(-2)^3 - 3\cdot(-2)^2 = 2\cdot(-8) - 3\cdot 4 = -16 - 12 = -28$

$T(-2, -28)$

$f'(-2) = 6\cdot(-2)^2 - 6\cdot(-2)^{\cancel{2}} = 6\cdot 4 + 12 = 36 = k_t$

$k_n = -\frac{1}{k_t} = -\frac{1}{36}$

<u>Enačba normale</u>: $y - y_1 = k_n(x - x_1)$

$y + 28 = -\frac{1}{36}(x + 2)$

$y = -\frac{1}{36}x - \frac{1}{18} + 28$

$y = -\frac{1}{36}x + \frac{503}{18}$

$6x^2 - 6x$

c) Prevojna točka: $f''(x) = 0$

$12x - 6 = 0$

$12x = 6 \,/ : 12$

$x = \frac{1}{2}$ $\quad y = -\frac{1}{2}$

$P\left(\frac{1}{2}, -\frac{1}{2}\right)$

Konveksna je na intervalu $\left(\frac{1}{2}, \infty\right)$.

Konkavna je na intervalu $\left(-\infty, \frac{1}{2}\right)$.

d) $y = 12x - 6 \longrightarrow k_t = 12$

$f'(x) = k_t = 12$

$6x^2 - 6x = 12$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x_1 = 2 \quad x_2 = -1$

O: Abscisi teh dveh točk sta $x_1 = 2$ in $x_2 = -1$.

6. $f(x) = \frac{x+1}{x^2}$

a) Ničla : $x = -1$

Pol : $x_{1,2} = 0$

Vodoravna asimptota : $y = 0$ oz. x os

Ekstremi : $f'(x) = 0$

$$f'(x) = \frac{(x+1)' \cdot x^2 - (x+1) \cdot (x^2)'}{x^4} = \frac{1 \cdot x^2 - (x+1) \cdot 2x}{x^4} =$$

$$= \frac{x^2 - 2x^2 - 2x}{x^4} = \frac{-x^2 - 2x}{x^4} = \frac{-x-2}{x^3}$$

$$f'(x) = 0 = \frac{-x-2}{x^3} \quad / \cdot x^3;\ x \neq 0$$

$0 = -x - 2$

$x = -2$

$y = \frac{-1}{4} = -\frac{1}{4}$

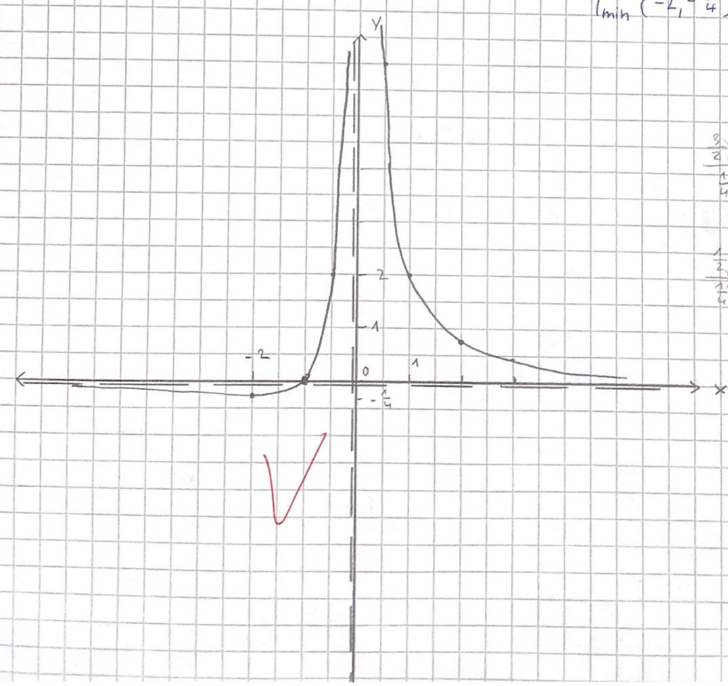
$T(-2, -\frac{1}{4})$

$\frac{1}{-3}$

$\frac{-1}{-1}$

x	-3	-2	-1
$f'(x)$	$-$	0	$+$

$T_{min}(-2, -\frac{1}{4})$



$\frac{3}{2}$ $\frac{1}{4}$

$\frac{1}{2}$ $\frac{1}{4}$

b) $[-3,-1]$ $\longrightarrow$ Iščemo globalni ekstrem. Ta zavzame ekstremno vrednost v lokalnem ekstremu ali v krajišču danega intervala.

Lok. ekstrem:

$T(-2, -\frac{1}{4})$

$T(-3, -\frac{2}{9})$

$T(-1, 0)$

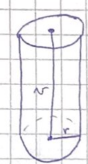
Globalni maksimum: $T(-1, 0)$ vrednost !

Globalni minimum: $T(-2, -\frac{1}{4})$

c) $f'(-1) = \dfrac{-(-1)-2}{(-1)^3} = \dfrac{-1}{-1} = 1 = \tan\alpha$

$\alpha = 45°$

7.



$V = 10\,\ell = \pi r^2 \cdot v \longrightarrow v = \dfrac{10\,\ell}{\pi r^2}$

P = ekstremna

$P = S_{pl} + S_{os}$ $\longrightarrow$ samo ena, saj je odprta posoda

$P = 2\pi r \cdot v + \pi r^2$

$P = 2\pi r \cdot \dfrac{10\,\ell}{\pi r^2} + \pi r^2$

$P = \dfrac{20\,\ell}{r} + \pi r^2$

$P_{(x)} = 20\ell \cdot r^{-1} + \pi r^2$

$P'(x) = -20\ell r^{-2} + 2\pi r = 0$

$-\dfrac{20\ell}{r^2} + 2\pi r = 0 \;/\cdot r^2$

$2\pi r^3 - 20\ell = 0$

$\pi r^3 = 10\,\ell$

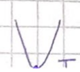
$r^3 = \dfrac{10}{\pi}$

$r = \sqrt[3]{\dfrac{10\ell}{\pi}} = 1{,}47\text{ dm}$

$v = \dfrac{10}{\pi r^2}$

$v = 1{,}47\text{ dm}$

r	1	1,47	2
$P'(x)$	$-$	0	+



O: Posoda bo imela najmanjšo površino, če bosta polmer ter višina enaka.