

$$1. a) y = x^3 \arcsin x$$

$$y' = 3x^2 \cdot \arcsin x + x^3 \cdot \frac{1}{\sqrt{1-x^2}} = 3x^2 \cdot \arcsin x + \frac{x^3}{\sqrt{1-x^2}} =$$

$$y' = x^2 \cdot \left( 3 \arcsin x + \frac{x}{\sqrt{1-x^2}} \right)$$

$$b) \int \frac{\sqrt{x} \sqrt{x}}{5 \sqrt{x-2}} dx = \int \frac{x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}}{x^{-\frac{1}{5}}} dx = \int x^{\frac{1}{4} + \frac{1}{2} + \frac{1}{5}} dx =$$

$$= \int x^{\frac{9}{20} - (-\frac{2}{5})} dx = \int x^{\frac{9+8}{10}} dx = \int x^{\frac{17}{10}} dx =$$

$$= \frac{10}{20} \cdot x^{\frac{27}{10}} + C$$

$$\int \frac{\sin x}{\cos^2 x} dx = \int \frac{2 \sin x \cos x}{\cos^2 x} dx = 2 \int \frac{\sin x}{\cos x} dx = 2 \int \tan x dx =$$

$$= 2 \cdot (-\ln |\cos x|) + C = \underline{\underline{-2 \ln |\cos x| + C}}$$

$$2. F(x) = \int (3x^2 - 2 \sin x + e^x) dx ; T(0,5)$$

$$\int 3x^2 dx + \int -2 \sin x dx + \int e^x dx = 3 \int x^2 dx - 2 \int \sin x dx + \int e^x dx =$$

$$= 3 \cdot \frac{1}{3} x^3 - 2 \cdot (-\cos x) + e^x + C =$$

$$= \underline{\underline{x^3 + 2 \cos x + e^x + C}} = F(x)$$

$$F(0) = 5 = 0^3 + 2 \cdot \cos 0 + e^0 + C$$

$$5 = 3 + C$$

$$\underline{\underline{C = 2}}$$

$$F(x) = \underline{\underline{x^3 + 2 \cos x + e^x + 2}}$$

$$3. f(x) = \sqrt{1-2x} - 3 = x^{\frac{1}{2}} - 3$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}} \cdot (-2) = \frac{1}{2} \cdot \frac{1}{\sqrt{1-2x}} \cdot (-2) = \frac{-1}{\sqrt{1-2x}}$$

$$f'(x) = \frac{-1}{\sqrt{1-2x}}$$

Niža:  $y=0$   $K_T = f'(-4) = \frac{-1}{\sqrt{1-2(-4)}} = \frac{-1}{3}$

$$0 = \sqrt{1-2x} - 3$$

$$3 = \sqrt{1-2x}$$

$$9 = 1-2x$$

$$2x = -8$$

$$x = -4$$

$$T(-4, 0)$$

Normala:  $y - y_1 = k_n(x - x_1)$

$$y - 0 = 3(x + 4)$$

$$y = 3x + 12$$

4.  $f(x) = \sin 3x + 4 \cos x$

a)  $f'(x) = 3 \cos 3x - 4 \sin x$

$$f\left(\frac{\pi}{6}\right) - f'\left(\frac{\pi}{6}\right) = 3 + 2\sqrt{3}$$

$$f\left(\frac{\pi}{6}\right) - f'\left(\frac{\pi}{6}\right) = \sin \frac{\pi}{2} + 4 \cdot \cos \frac{\pi}{6} - (3 \cos \frac{\pi}{2} - 4 \cdot \sin \frac{\pi}{6}) =$$

$$= \sin 90^\circ + 4 \cdot \cos 30^\circ - 3 \cos 90^\circ + 4 \cdot \sin 30^\circ =$$

$$= 1 + 4 \cdot \frac{\sqrt{3}}{2} - 3 \cdot 0 + 4 \cdot \frac{1}{2}$$

$$= 1 + 4 \cdot \frac{\sqrt{3}}{2} - 3 \cdot 0 + 4 \cdot \frac{1}{2} = 1 + 2\sqrt{3} + 2 = 3 + 2\sqrt{3}$$

b)  $f'\left(\frac{\pi}{6}\right) = 3 \cos \frac{\pi}{2} - 4 \sin \frac{\pi}{6} = 3 \cdot \cos 90^\circ - 4 \cdot \sin 30^\circ =$

$$= 3 \cdot 0 - 4 \cdot \frac{1}{2} = -2 = k_t$$

Funkcija u točki zj. abscise  $x_0 = \frac{\pi}{6}$  pada, jer je omerni koeficijent tangente u točki negativan.

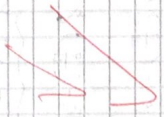
$f' < 0$

11/11/2021

5.  $f(x) = (2x-1) \cdot e^{3x}$   
 $f'(x) = 2e^{3x} + (2x-1) \cdot 3 \cdot e^{3x}$   
 $f'(x) = 2e^{3x} + 6xe^{3x} - 3e^{3x}$   
 $f'(x) = e^{3x}(2+6x-3) = e^{3x}(6x-1)$   
 $f''(x) = 3e^{3x}(6x-1) + e^{3x} \cdot 6$   
 $f''(x) = 18xe^{3x} - 3e^{3x} + 6e^{3x}$   
 $f''(x) = e^{3x}(18x+3)$

Prevojna točka:  $f''(x) = 0$   
 $e^{3x}(18x+3) = 0$   
 ①  $e^{3x} = 0$  ②  $18x+3 = 0$   
 $18x = -3 / :18$   
 $x = -\frac{1}{6}$

$P(-\frac{1}{6}; -0,81)$



Konkavna funkcija:  ~~$x \in (-\infty, -\frac{1}{6})$~~   
 $x \in (-\infty, -\frac{1}{6})$

Konveksna funkcija:  $x \in (-\frac{1}{6}, \infty)$

x	-1	0
f'(x)	-1	0

6.  $x^2 - 2y^2 = 2$

Odnos:  $2x - 4yy' = 0$

$4yy' = 2x$   
 $y' = \frac{2x}{4y} = \frac{x}{2y}$

$\frac{y'}{y} = k = 1 = \frac{2x}{4y}$



$x^2 - 2y^2 = 2 / :2$

$\frac{x^2}{2} - \frac{y^2}{1} = 1$

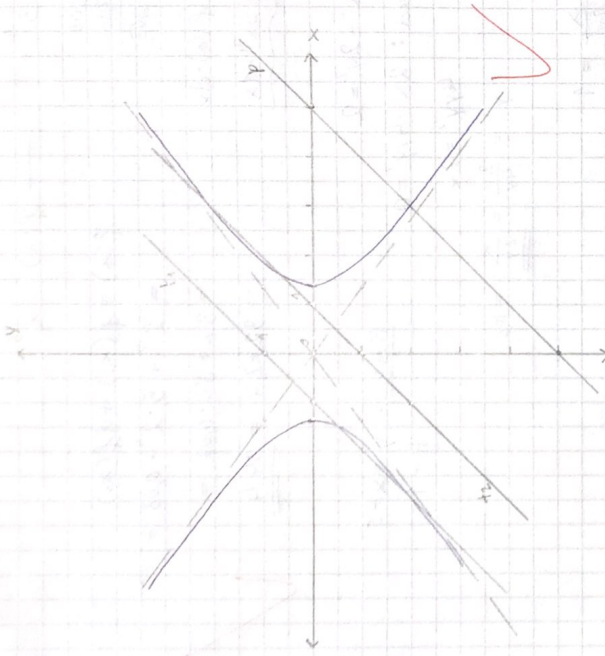
$a = \sqrt{2}$   
 $b = 1$

Pravica:  $x - y = 5$

$y = x - 5$   
 $k_1 = 1$

~~Pravica~~

6. naloga na drugem listu + graf



7.  $f(x) = 3 \ln \left( \frac{2x}{x-1} \right) = 3 \ln z ; z = \frac{2x}{x-1}$

$D_f: \frac{2x}{x-1} > 0$

$x_1 = 0$   
 $x_2 = 1$

$D_f: x \in (-\infty, 0) \cup (1, \infty)$

$f'(x) = 3 \frac{1}{z} \cdot z' = \frac{3}{\frac{2x}{x-1}} \cdot \left( \frac{2 \cdot (x-1) - 2x \cdot 1}{(x-1)^2} \right) =$

$= \frac{3 \cdot (x-1)}{2x} \cdot \frac{2x-2-2x}{(x-1)^2} = \frac{3}{2x} \cdot \frac{-2}{x-1} = \frac{-6}{2x \cdot (x-1)}$

Nicta:  $3 \ln \left( \frac{2x}{x-1} \right) = 0$

$\frac{2x}{x-1} = 1$

$2x = x-1$   
 $x = -1$

$T(-1, 0)$

$f'(-1) = \frac{-6}{-2 \cdot (-2)} = \frac{-6}{4} = -\frac{3}{2}$

$-\frac{3}{2} = \tan \alpha$

$\alpha = -56^\circ 19' + 180^\circ$

$\alpha = \underline{\underline{123^\circ 41'}}$

8.  $2,03^5 = ? \rightarrow f(x) = x^5$

$x = 2,03$   
 $x_0 = 2,00$   
 $dx = 0,03$

$f(x) = f(x_0) + df(x_0)$   
 $= 2^5 + 5 \cdot 2^4 \cdot 0,03 =$   
 $= 32 + 80 \cdot 0,03 =$   
 $= 32 + 2,4 = \underline{\underline{34,4}}$

✓ 10

6.  $x^2 - 2y^2 = 2$

Calvod:  $2x - 4yy' = 0$

$4yy' = 2x$

$y' = \frac{2x}{4y} = \underline{\underline{\frac{x}{2y}}}$

$\frac{x}{2y} = 1$

$x = 2y$

$(2y)^2 - 2y^2 = 2$

$4y^2 - 2y^2 = 2$

$2y^2 = 2$

$y^2 - 1 = 0$

$(y-1)(y+1) = 0$

$y_1 = 1 \quad y_2 = -1$

$x_1 = 2 \quad x_2 = -2$

Premica:  $y = x - 5$   
 $k = 1$

$T_1(-2, -1)$

$T_2(2, 1)$

✓

tangentna:  $y - y_1 = k_f(x - x_1)$

1.  $y + 1 = 1 \cdot (x + 2)$

$y = x + 1$

2.  $y - 1 = 1 \cdot (x - 2)$

$y = x - 1$

✓

✓